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# Public report on the wavelet-based simulation technique

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# 1 Introduction

An analog circuit simulator is an indispensable part of any electronic design automation (EDA) toolbox, since it enables prediction of a complex circuit behavior *before* the circuit is actually produced. This in turn results in the unprecedented manufacturing cost reduction, thus making possible the development of very complex and highly integrated electronic devices, which revolutionized our life- and work-style in the last 20 years.
The ever-increasing operating speed of electronic devices along their ever-shrinking size, brought devices such as mobile phones into our everyday routine. Moreover, today we witness increasing demand on even faster and smaller devices, that can facilitate various new applications and business opportunities, e.g., mobile television or super-fast Internet access anytime, any place. These new demands from both private and business users put extreme pressure and raise new challenges for existing circuit simulation techniques.
One particular challenge for modern simulation tools is the increased amount of digital-like content for practically all modern analog circuit designs, i. e., existence of signals with very steep slopes across all application areas. Existing algorithms for both the time-domain (e. g.,transient analysis) and the frequency-domain analog circuit simulations (e. g., Harmonic Balance analysis) exhibit serious problems when attempting to deal with increasingly mixed analog/digital designs. To address this challenge, we propose the usage of wavelets in the framework of the circuit simulation. Our aim is to represent digital-like signals more compactly in order to enable both quantitative and qualitative improvements in the simulation within this class of problems.
After identifying the above-mentioned bottlenecks, the EU-funded project ICESTARS was setup in 2008 with a focus on the development and implementation of algorithms for the simulation of radio frequency (RF) and mixed analogue/digital circuits for the next generation of mobile radio ICs in the frequency range far beyond 1 GHz. In this Report, we describe a novel wavelet-based algorithm for analog circuit simulation, which we developed and tested within an existing industrial in-house circuit simulator.

# 2 The analog circuit simulation problem

In order to analyze an electronic circuit and predict its behavior, we consider circuit description in the charge/flux oriented modified nodal analysis (MNA) formulation, which yields a mathematical model in the form of an initial-value problem of differential-algebraic equations (DAEs):

$$\frac{d}{dt}\boldsymbol{q}\big(\boldsymbol{x}(t)\big) + \boldsymbol{f}\big(\boldsymbol{x}(t)\big) = \boldsymbol{s}(t). \tag{1}$$

Here $\boldsymbol{x}$ is the vector of node potentials and specific branch voltages and $\boldsymbol{q}$ is the vector of charges and fluxes. Vector $\boldsymbol{f}$ comprises static contributions, while $\boldsymbol{s}$ contains the contributions of independent sources. The goal of an circuit simulator technique is to numerically solve (1) within prescribed error tolerances and as efficiently as possible. For this purpose, we propose the usage of wavelets.

## 3 Wavelets

Wavelet theory emerged during the 20th century from the study of Calderon-Zygmund operators in mathematics, the study of the theory of subband coding in engineering and the study of renormalization group theory in physics. The common foundation for the wavelet theory was laid down at the end of the 80's and beginning of the 90's by work of Daubechies, Morlet and Grossman, Donoho, Coifman, Meyer, Mallat and others. Today wavelet-based algorithms are already in productive use in a broad range of applications, such as image and signal compression (JPEG2000 standard, FBI fingerprints database), speech recognition, numerical analysis (solving operator equations, boundary value problems), stochastics, smoothing/denoising data, physics (molecular dynamics, geophysics, turbulence), medicine (heart-rate and ECG analysis, DNA analysis) to name just a few. For more details, we refer an interested reader to the exhaustive literature on wavelets and their applications.

Recent approaches to the problem of multirate envelope simulation indicate that wavelets could also be used to address the qualitative challenge by the development of novel wavelet-based circuit simulation techniques capable of the efficient simulation of a mixed analog-digital circuit [1]. Our work within Task 1.1 of the ICESTARS WP1 has confirmed that this is indeed the case.

We start from the wavelet expansion of a function $f$, given as

$$f = \sum_{k\in\mathcal{I}} c_k\,\phi_k + \sum_{j=0}^{\infty}\sum_{k\in\Lambda_j} d_{jk}\,\psi_{jk}. \tag{2}$$

Here, $j$ refers to a level of resolution, while $k$ describes the localization in time or space, i.e., $\psi_{jk}$ is essentially supported in the neighborhood of a point $x_{jk}$. The wavelet expansion can be seen as coarse scale approximation $\sum_{k\in\mathcal{I}} c_k\,\phi_k$ by the scaling functions $\phi_k$ complemented by detail information of increasing resolution $j$ in terms of the wavelets $\psi_{jk}$.

Since a wavelet basis consist of an infinite number of wavelets one has to consider approximations of $f$ by partial sums of the wavelet expansion (2). Usually only those wavelets, which carry important detail information indicated by large wavelet coefficients, are added to the coarse scale part $\sum_{k\in\mathcal{I}} c_k\,\phi_k$. Using wavelet bases with appropriate regularity, stability and localization properties, this approach leads to the very efficient representation of signals with 'isolated singularities', e.g., jumps or steep edges.

## 4 An Adaptive Wavelet Galerkin Method

To facilitate numerical computations, we approximate the solution of the circuit equations (1) as a finite wavelet expansion

$$\boldsymbol{x}_\varepsilon(t) = \sum_{(j,k)\in\Lambda_\varepsilon} \boldsymbol{d}_{jk}\,\psi_{jk}(t).$$

Here, the finite index set $\Lambda_\varepsilon$ shall be chosen in order to achieve the required accuracy with a minimal number of wavelets. Now, we have to determine a finite set of vector valued coefficients $\boldsymbol{d}_{jk}$, instead of the vector valued function $\boldsymbol{x}(t)$. Since we want an approximation to the solution

of the circuit equation, we have to impose conditions on $\boldsymbol{d}_{jk}$, which ensure that $\boldsymbol{x}_\varepsilon(t)$ satisfies the circuit equations (1), at least approximately.
This can be done by the Petrov-Galerkin discretization

$$\int_0^T \Big(\frac{d}{dt}\boldsymbol{q}\big(\boldsymbol{x}(t)\big) + \boldsymbol{f}\big(\boldsymbol{x}(t)\big) - \boldsymbol{s}(t)\Big)\,\theta_\ell\,dt = 0, \qquad \ell = 1,\dots,n. \tag{3}$$

Integrating the circuit equations against $n$ test functions $\theta_\ell$, we obtain a system of nonlinear equations, which can be used to determine the coefficients $\boldsymbol{d}_{jk}$. To obtain a well-posed problem we set $n$ equal to the cardinality of $\Lambda_\varepsilon$, while the $\theta_\ell$ are chosen in accordance to the wavelet system determined by $\Lambda_\varepsilon$.
The nonlinear system (3) is solved by Newton's method, an iteration method, where we improve the solution step by step by solving a linear system obtained by a linear approximation of (3). These iteration steps are used also, to improve the index set $\Lambda_\varepsilon$ based on information of the current approximation. That is, we start with a coarse approximation, which is improved step by step on a grid of increasing size.
This approach has two benefits. First, we start with a small wavelet system reducing the computational cost for the iteration steps in the beginning, while the final largest wavelet set is only used in the final iteration steps. Furthermore, the final wavelet set is well adapted to the solution, since it is updated based on the improved information we obtain during the iteration process.

## 5 Validation of the wavelet method

A prototype of the wavelet algorithm has been implemented and tested within the framework of a productively used in-house circuit simulator and tested on a variety of RF circuits. We were able to reproduce the results of a well-established and highly-accurate transient analysis from the same simulator up to a very high precision, well beyond standard accuracy requirements [2, 3]. Here it should be noted that the existing transient analysis has been continuously optimized over the period of more than 20 years. With the new wavelet algorithm implementation we were able to obtain already very good results after only 2 years of development. Moreover, we have already identified the possibilities for further optimization of the existing algorithm and its implementation.
For example, Fig. 1 shows the input resp. output signal of a single-ended down-converting Gilbert mixer, whose purpose is to convert fast RF signal with frequency of 0.95 GHz to slower baseband signal with frequency 50 MHz. The dashed line is the time-domain waveform produced by wavelet method, superimposed on an simulation result of the standard transient analysis (solid line). One can see that they match almost perfectly both for digital-like input signal and smoother output signal, thus confirming the capability of the wavelet-based algorithm to correctly capture mixed analog/digital behavior of a modern RF circuits.
The grid size and CPU time comparison for this particular test example is shown in Fig. 2. They are similar to the other simulation comparisons performed on RF circuits from Project's validation test suite. One can see that for increasing accuracy a much smaller grid is needed for our wavelet method compared to the standard transient analysis [2, 3]. Unfortunately, at the present stage of the implementation this advantage in the grid size does not result in a lower

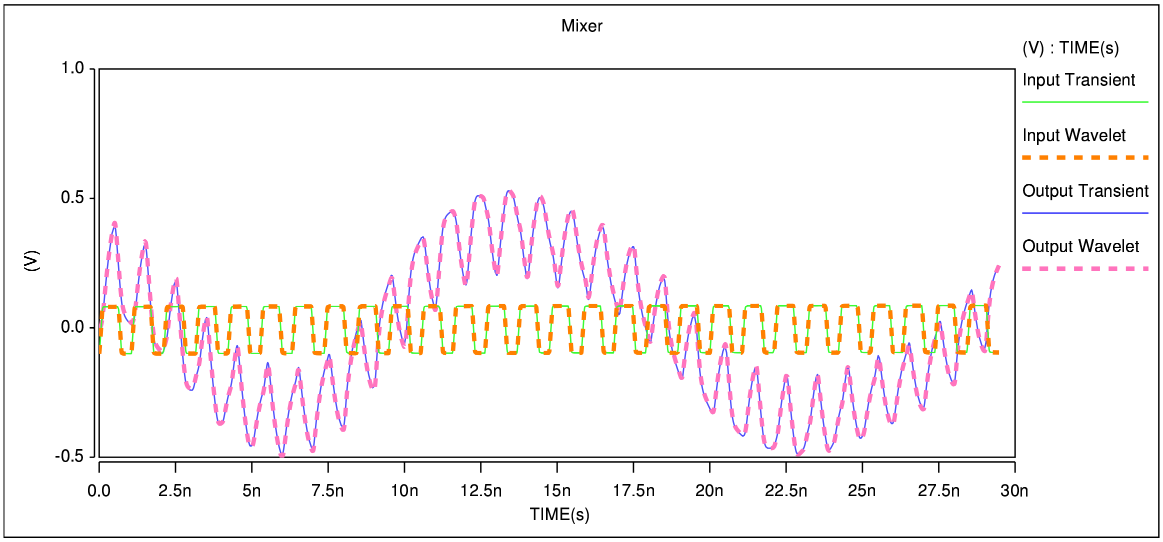


Figure 1: Time domain output signal for a mixer.

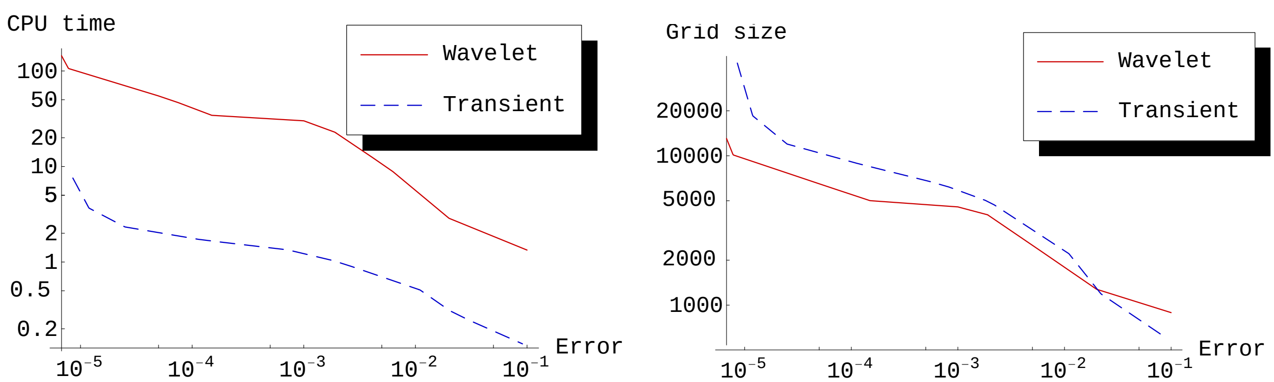


Figure 2: Simulation results for the mixer. Computation time versus error (bottom left), and grid size versus error (bottom right) for transient analysis and adaptive wavelet analysis.

computation time, although we are already close to the performance of the highly-optimized transient analysis.

# 6 Conclusion

To summarize, during the ICESTARS project a novel wavelet-based technique for mixed analog-digital circuit simulation has been developed and the first completely functioning prototype of the wavelet-based algorithm has been implemented within an industrial in-house circuit simulator. In doing so, we sucessfully fulfilled Project goals set for the Task 1.1.
Since we see several additional opportunities to further improve both the algorithm as well as the software implementation itself, we are optimistic that wavelet analysis will become a

valuable tool for circuit simulation in the EDA toolbox of the future. Therefore, it is envisaged that the further optimization of this wavelet-based method for circuit simulation will continue at Infineon beyond the ICESTARS duration.

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