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Public report on the multirate wavelet and envelope techniques

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Contents

1 Introduction

This workpackage "‘Time-Domain Techniques"’ of the ICESTARS project 2008-2010 is focussed on the development and implementation of algorithms for the time-domain simulation of radio frequency (RF) and mixed analogue/digital circuits for the next generation of mobile radio ICs in the frequency range far beyond 1 GHz. In what follows the simulation problem is first described for an interested community where only basic knowledge of electronic circuits, communications and signal processing as well as numerical techniques are required. Afterwards, the novel techniques and treatments of the simulation problem are explained employing simulations achieved with the novel tools.

2 The simulation problem

Electronic circuits consist of devices which are connected via ports or nodes. The connections form the topology of the circuit which can be described by simple matrices with matrix entries 0 or ± 1 related to the connections (nodes) and circuit branches. Any branch is incident to a pair of nodes. One circuit node is the reference node or ground node with node voltage zero. All other node voltages a related to this reference node. Moreover, to any circuit branch a current is related, the branch current. A branch voltage is the voltage drop along the branch which is simply the voltage difference of the incident nodes. The circuits fulfill the well-known Kirchhoff's current and voltage laws (KCLs, KVLs), i.e. the sum of branch currents entering a node and the sum of voltage drops of a loop of branches are identically zero, respectively.

The relations between the branch currents and voltages depend on the physical devices. Linear devices such as resistors, capacitors, inductors and transformers can be handeled easily by so called device equations relating the currents and voltages by simple linear equations. Semiconductors such as MOS and bipolar transistors and diodes exhibit however very complicated nonlinear (ordinary differential) equations, often several hundreds for highly accurate models. These models are referred to as lumped or concentrated elements, this means that electromagnetic effects are concentrated within the device and do not affect others. Another class are the distributed devices such as transmission lines. Distributed devices are formulated by partial differential equations which can also be both linear and nonlinear. It is important to mention that all electromagnetic effects are fully concentrated within the device and that there is no electromagnetic coupling or crosstalk between these devices. Only under these assumptions the Kirchhoff laws are valid. It shall be mentioned here that WP3 deals with the electromagnetic/-circuit co-simulation, i.e. some devices such as on-chip spiral inductors are modelled by their electromagnetic field equations and incorporated into a circuit environment.

Having a circuit topology and the device constitutive equations at hand, one can set up a system of equations to be solved by hand or by a computer. Several techniques were investigated and the most prominent one is the modified nodal analysis or in short MNA. The nodal analysis works as follows: using the node voltages as the unknown variables the KVLs are automatically fulfilled, reducing therefore the amount of equations and hence computer time. A circuit exhibiting $N+1$ nodes including the ground node has N linearly independent KCLs. The KCL for the ground node is the negative sum of all the other KCLs. Hence N linear independent KCLs are to be fulfilled. Inserting into the KCLs the device constitutive equations gives a system of N equations. Because practically all circuits include dynamical elements such as capacitors and inductors one obtains N ordinary differential equations or differential-algebraic equations (DAEs) in time. It pointed out that pure nodal analysis is not flexibel enough for modelling circuit devices, because it presupposes a formulation referred to as admittance form. For handling circuit devices in so-called admittance form a variant has been developed which is referred to as modified nodal analysis and is the standard method in most circuit simulators.

The ordinary differential-algebraic system in time must now be solved by a computer after stimulus signals have been applied to the circuit. Only for some special cases such as linear circuits with a harmonic stimulus the solution can be evaluated in closed form. Instead one has to resort to numerical techniques referred to as numerical integration methods. Basically, numerical techniques discretize the DAEs, this means that a solution is not calculated over the continuous time but at distinct time points instead. This is referred to the discretization grid or mesh. The difference between the time points is referred to as the time step which is in most cases a variable to exploit latencies of the circuit. It is easy to see that these methods are efficient as long as the solution, the unknown voltages and currents waveforms, behave smoothly in time. This means in the contrary that the spectrum of the waveforms exhibits a small bandwidth. On the other hand, waveforms with a huge bandwidth require small time steps and hence a huge simulation time. This is in agreement with the celebrated Nyquist's theorem, which says roughly spoken that the discretization and the signal bandwidth is related, the higher the bandwidth, the smaller the discretization steps.

Circuits with a huge bandwidth of their waveforms typically occur in radio frequency applications such as mobile phones. From the above, it is clear that these circuits require a large simulation time even for a moderate a circuit size.

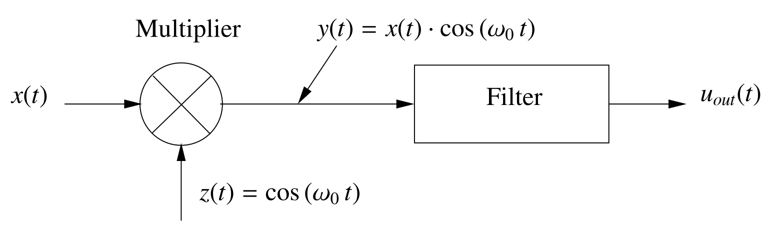

Figure 1: Block diagram of a simple transmitter.

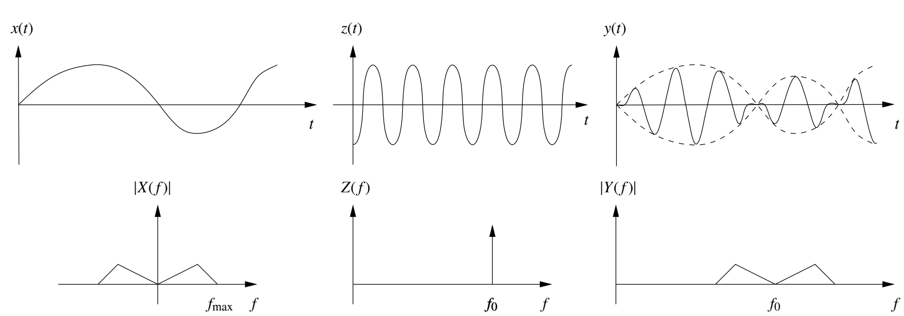

Figure 2: Amplitude modulation: signal representation in the time domain (top) and frequency domain (bottom).

2.1 Radio Frequency circuits and signals

The Fig. 1 shows a block diagramm of a transmitter. The information signal which is referred to as baseband signal or envelope is modulated by a mixer circuit which is stimulated by an oscillator with carrier or center frequency f_0. The Fig. 2 exhibits the signals and the frequency spectrum of the signals involved. The information signal has a small bandwidth, in typical applications in the range of a few kHz to MHz depending on the application. For example the widespread GSM mobile phone system employs signals of 200 kHz bandwidth. On the other hand, the center frequency of the transmitted signal is around 920 MHz for GSM. Hence, the ratio of the signal bandwidth of the unmodulated baseband signal to the carrier frequency is very high. This ratio will even become larger for the next generations of mobile phone systems with center frequencies in the GHz range. The spectrum of the transmitted signal is depicted on the right of Fig. 2. Such a spectrum is referred to as a bandpass spectrum. This means that the spectrum of the signal or equivalently the signal power is concentrated within a bandwidth around a carrier frequency. Outside this frequency range, the signal power is negligible. This fact can be used to speed up the simulation process significantly. Using standard simulation tools this sparsity of the spectrum cannot be exploited. The allowed time steps of a numerical integration technique of the system of differential equations is fixed by Nyquist's theorem, because conventional methods cannot exploit the sparsity of the spectrum encountered in typical RF circuitry.

3 Solving the Radio Frequency problem: the multirate approach

3.1 Taking a glimpse on the multirate method

The novel tools developed within the ICESTARS project however cope with this in a unique way as discussed next. Consider the signal waveform depicted in Fig. 3. The smooth sinusoidal envelope is modulated by a sinusoidal carrier. For illustrative reasons the ratios of the frequencies are here only 1:10. The idea behind the novel technique is to replace the time t by two distinct times t_1, t_2 with different time scales of ratio 1:10 in this example. One obtains the 2-dimensional representation of the signal as shown in Fig. 4. Because sinusoidal waveforms are involved, the 2-dimensional waveforms can be expanded periodically along the axes. The original signal of Fig. 3 is reconstructed along the curve $t_1 = t_2 = t$. One says that the original waveform is embedded into a more general signal $x(t_1,t_2)$. A discretization of the general waveform is now very easy, because the waveform is smooth along the axes and hence overcomes the Nyquist's theorem.

A similar interpretation can be given in the frequency domain. The spectrum of a typical mixer output is illustrated in Fig. 5. As already discussed, the carrier frequency f_0 is in practical cases orders of magnitudes larger than the bandwidth of the baseband signal and one cannot exploit the sparsity of the spectrum. By introducing the generalized waveform $x(t_1,t_2)$ its Fourier spectrum is illustrated in Fig. 5. The 2-dimensional spectrum is now compact with equidistant spacings along the frequency axes. The different time scales of the envelope and the carrier are reflected by the different axis scales. It can be proven that this concept can be used to simulate the RF circuits efficiently.

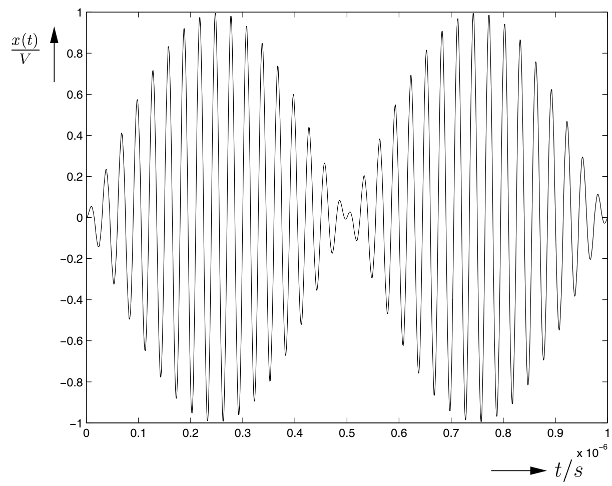

Figure 3: Output voltage of a mixer circuit.

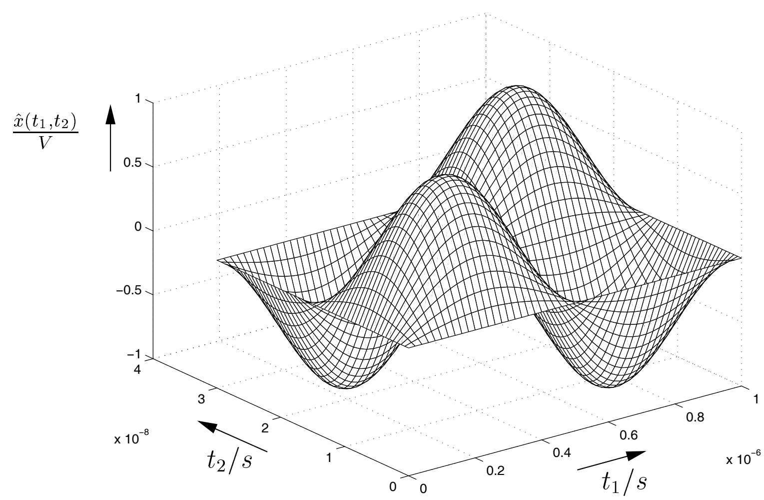

Figure 4: 2-dimensional representation of the output voltage of a mixer circuit.

3.2 The multirate approach

After taking a glimpse on the method in a heuristic fashion a deeper lock into the technique is now falling due. As mentioned above, MNA leads to a system of differential equations of the form

$$f(x(t), \dot{x}(t), t) = i(x(t)) + \frac{\mathrm{d}}{\mathrm{d}t} q(x(t)) + b(t) = 0, \quad x(0) = x_0 \tag{1}$$

wherein $x \in \mathbb{R}^N$ is the vector of unknowns, $i, q \in C^1(\mathbb{R}^N, \mathbb{R}^N)$ are the vectors of currents entering the nodes and charges and fluxes, respectively and $b \in C^1(\mathbb{R}, \mathbb{R}^N)$ is a stimulus vector, both voltages and currents. Finally t is the time. As illustrated above, the idea behind the novel technique is to split the signal into a fast time scale, the carrier, and a low time scale, the envelope. One replaces therefore the time t in the equation above by two times (t_1, t_2) and

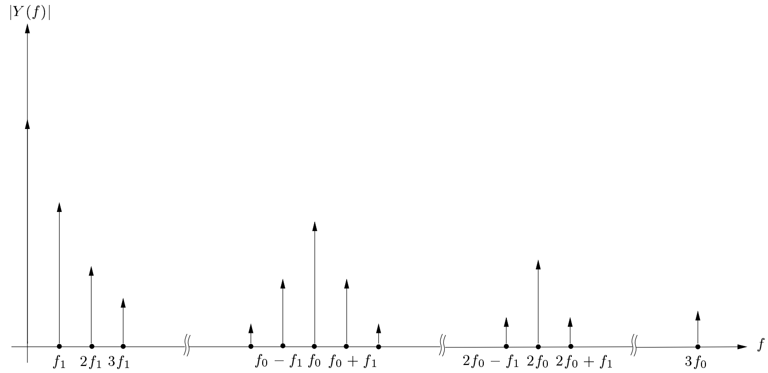

Figure 5: Typical spectrum of a mixer circuit.

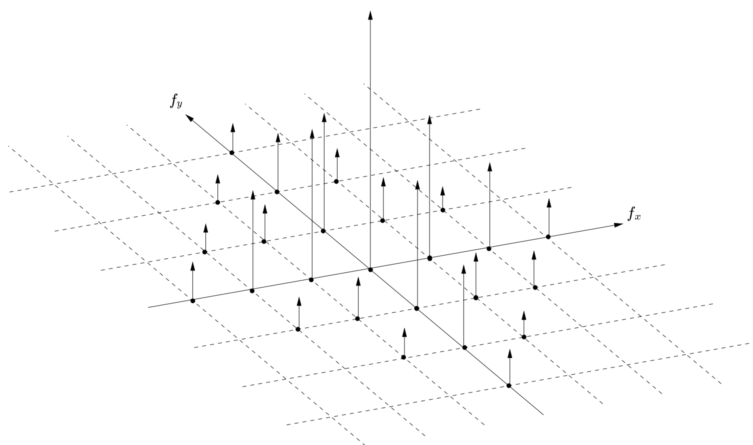

Figure 6: Demonstration of the embedding technique in the frequency domain.

obtains

$$\hat{f}(\hat{x}(t_1,t_2),\nabla\hat{x}(t_1,t_2),(t_1,t_2)) = \\ i(\hat{x}(t_1,t_2)) + \sum_{i=1}^{2}\omega_i\frac{\partial}{\partial t_i}q(\hat{x}(t_1,t_2)) + \hat{b}(t_1,t_2) = 0 \tag{2}$$

Firstly, the waveform $x(t)$ in the arguments of f, i and q are replaced by the generalized waveform $\hat{x}(t_1,t_2)$. Moreover, the time derivative $\frac{\mathrm{d}}{\mathrm{d}t}$ is replaced by the partial derivatives $\sum_{i=1}^{2}\omega_i\frac{\partial}{\partial t_i}$. Hence (2) is a partial differential instead of an ordinary differential equation. It can be proven that the solution of the ordinary DAE (1) is obtained by simply setting $x(t) = \hat{x}(t_1 = \omega_1 t, t_2 = \omega_2 t)$. Mathematicans refer to this curve as the characteristic curve of the partial differential equation. It shall be noted, that the formulation of the partial differential equation differs for different applications. The example here is referred to as the so called 2-tone steady state simulation which plays an important role in the characterization of RF circuits, i.e. for evaluating the second and third order intercept point IP2, IP3, respectively. This concept can be generalized to a multitone test, i.e. employing stimulus waveforms with more than 2 fundamental frequencies. Another application is considered in the next subsection.

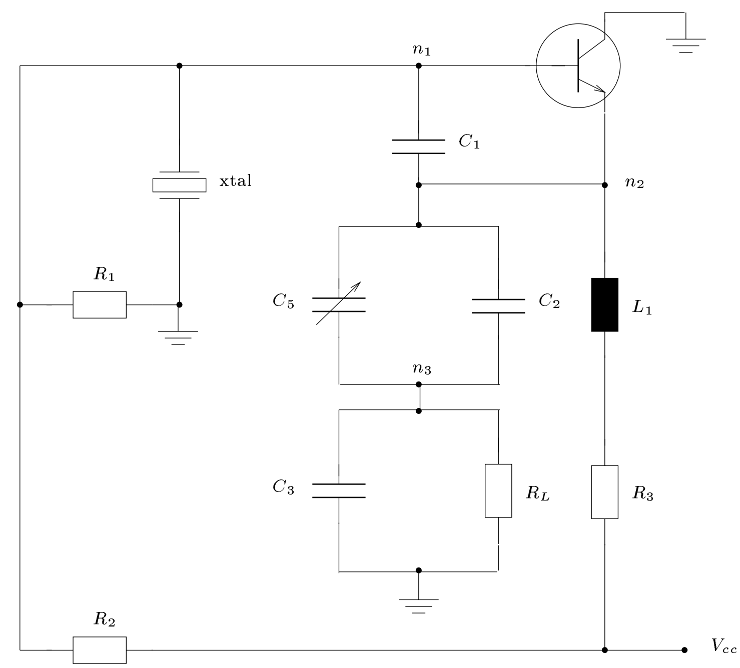

Figure 7: Schematic of a 3 MHz Colpitts quartz crystal oscillator.

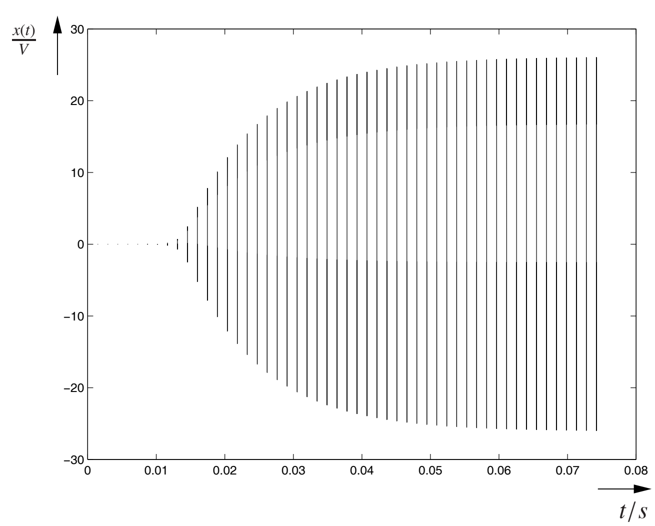

Figure 8: Initial transient response of a quartz crystal oscillator. Each line represents a single oscillation.

The resulting partial differential equations exhibit mixed boundary and initial value conditions. The oscillatory behavior leads to a boundary condition whereas the envelope results into an initial value problem. The PDE can be solved by either finite difference or Petrov-Galerkin type techniques. Task 1.2 employs a Galerkin approach based on a wavelet basis whereas the focus on task 1.3 is on novel finite difference schemes which specific emphasis on oscillatory signals.

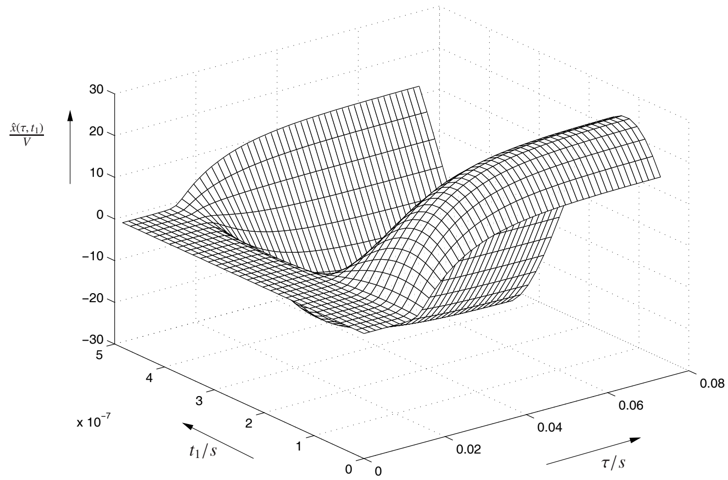

Figure 9: Bivariate solution of the PDE (2) for one waveform of the quartz crystal oscillator.

3.3 Transients of quartz crystal oscillators

Quartz crystal oscillators play an important role in the design of RF circuits because they are used as a time reference. Due to the high quality factor Q of the crystal the frequency of the oscillator is highly stable and can hardly affected by noise. Roughly spoken, the higher the Q factor the better the frequency stability. A schematic of a quartz crystal oscillator is depicted in Fig. 7. Though the circuit is very small, it is nevertheless hardly to simulate. This has to do with the high quality factor which leads to a very slow transient response. This is illustrated by Fig. 8. Each line represents one oscillation of the 3 MHz oscillator. As one can see from the scale that the initial transient response is about 70 ms. Knowing this number is very important for a circuit designer because for times below this figure the RF circuit is not operational. Also here we face the situation of a highly smooth envelope which contrasts with the oscillator frequency of 3 MHz in this case. Unlike the examples above, the different time scales are not caused by the stimulus signals but rather is a feature of the circuit itself, the high quality factor in this case. These circuits are referred to as stiff and the corresponding differential equations as stiff differential equations. Stiff differential equations are hard to simulate and require huge run times even for differential equations of moderate size. Moreover the unavoidable numerical error caused by the discretization can hardly be controlled. Based on similar ideas as described above the system of stiff ordinary differential equations can be reformulated by a system of partial differential equations. The result for the depicted oscillator is shown in Fig. 9. Again the waveform has been generalized to a signal with a slowly varying envelope in the τ scale which reflects the transient response of the circuit on the one hand and the oscillation of the circuit in the t_1 direction, reflecting the oscillation. Please note the different time scales along the axes. The waveform shall be considered as periodically repeated in the t_1 direction. When one compares this figure with Fig. 8 one can see that the envelope along the τ axis corresponds to the envelope obtained by solving the underlying system of ordinary DAEs. Due to the high center frequency, the plot cannot resolve the oscillations. Due to Nyquist's theorem, the time steps of the classical solver must be smaller than the period of oscillation which restricts the step size and makes the simulation burdensome. However a quasi sinusoidal waveform can be represented by only a few discretization points. The same is valid for a smooth envelope. The 2-dimensional signal of Fig. 9 makes use of this fact. According to Nyquist's theorem which can now be applied independently in the τ and t_1 direction an individual step size can be applied along the different scales.

3.4 Wavelet method for periodic steady state

We modified the basic wavelet-based method such that it is capable of computing the periodic steady state solution (PSS) using periodic B-splines and wavelets.

To compute the PSS of an oscillating circuit one has to solve a periodic boundary value problem for the circuit equations, i.e., the periodicity condition

$$x(t) = x(t+T) \tag{3}$$

replaces the initial condition $x(t_0) = x_0$. This can be achieved by directly enforcing the boundary conditions through additional equations as it is done in the shooting method. Another approach is to expand the solution in a periodic basis and determine the expansion coefficients as a solution of discretized circuit equations, e.g., a Fourier basis in Harmonic Balance.

Next, to enable the computation of a PSS via the adaptive wavelet method, a basis of p-periodic wavelets is needed. This basis can be generated by a standard approach from a wavelet basis. For the non-uniform spline wavelets used in our approach, this can be ensured by a periodicity condition on the spline knots, while the convergence of the infinite sum follows from the compact support of the wavelets.

Finally, for computation of PSS we have to distinguish two cases.

1. In a **driven circuit** an independent periodic source with a common period T is present, i.e., $g(x,t) = g(x,t+T)$. Here the period length is determined by the definition of the sources and is therefore known in advance.

2. An **autonomous circuit** contains only (constant) DC-sources, i.e., $g(x,t) = g(x)$. Here the oscillatory behavior depends on the particular design of the circuit, and the period length T is not known in advance and thus has to be determined during the simulation.

In both cases the problem of the PSS computation is solved by expanding the solution x in a periodic basis, i.e., $x(t) = \sum_k c_k \varphi_k(t)$, where $\varphi_k(t) = \varphi_k(t+T)$. A periodic discretization is then obtained, if a condition $\tau_n - \tau_0 = T$ is imposed. For the case of a driven circuit, the earlier developed adaptive wavelet method can be applied, if we have periodic spline bases as well as the corresponding spline wavelets for grid adaption.

For autonomous circuits a period of oscillation T has to be determined as an additional unknown. Since for any T-periodic solution $x(t)$ of the autonomous system $x(t-T)$ is also a solution, the uniqueness of solution has to be enforced. This is done by imposing an additional equation, the so called phase condition. Thus, the number of unknowns equals again the number of equations.

One of the main development difficulties with the realization of this method was the implementation of the fast wavelet transform needed for the wavelet based grid adaption. Although, the above periodization approach leads to a straight forward derivation from the non-periodic wavelet transform, the implementation is challenging since one has to deal with finite representations of infinite periodic sequences.

The routines for periodic splines and wavelets form the core of the wavelet envelope method, where similar periodic boundary problems have to be solved.

3.5 Wavelet envelope algorithm using periodic wavelets

Discretizing the multirate formulation 2 in the slow time scale t_1 by a multi-step method one has to solve a periodic boundary problem for each time step. In the wavelet envelope algorithm we solve this problem using the periodic wavelet solver from Sect. 3.4.

We discretize in t_1 by a Rothe method using a linear multi step method and obtain the periodic boundary value problem in the t_2 time direction. The wavelet envelope method has been implemented using GEAR-BDF or Trapezoidal rule for the discretization in the slow time scale t_1. Some modifications of the wavelet periodic steady state algorithm had to be made, due to the discretization in this slow time scale direction. In each time step the wavelet grid is updated for an optimal adaptive representation of the fast carrier signal. Furthermore, the algorithm is adaptive in the slow time scale too, by a step size control of the multi step method.

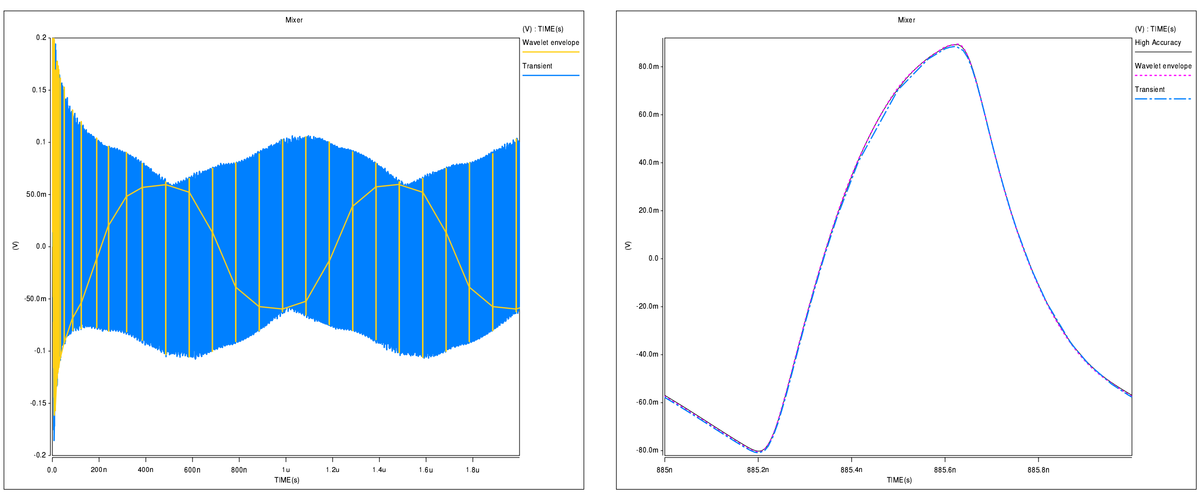

Figure 10: Wavelet envelope simulation of the mixer compared to a transient analysis (total view and detail).

Fig 10 shows the wavelet envelope simulation results for the c9linmix mixer circuit (cf. D5.2) compared to transient analysis results. As can be seen both envelope and the detail view show good agreement. The wavelet envelope algorithm was successfully tested on the amplifier and mixer example from the validation test suit, see D5.3 for details. The validation results show that the robustness and accuracy of the algorithm are entirely achieved, while efficiency, although satisfying for the first prototype has still a lot of potential for further improvements.

3.5.1 Transients of mixer circuits

Mixers are another typical example for multirate signals. In a receiver the radio frequency or bandpass signal is mixed to the baseband or a low intermediate frequency, i.e. a typical example of a multirate problem. Several mixer circuits have been simulated for validation of the PDE approach for this class of circuits, including a Diode mixer, a single balanced mixer, a Gilbert cell mixer and a folded-switching mixer. The circuit designs have been taken from state-of-the-art PhD Theses and publications in the IEEE Journal of Solid-State Circuits. The mixer examples pointed out that variants of the method are required for the mixers used in an uplink or downlink scenario. The reason, which has been not foreseen at the beginning, is that in the downlink two independent RF signals (oscillator and antenna signal) of different frequency are involved from which the baseband signal is derived. Contrary, in an uplink scenario, the baseband signal and the high-frequency oscillator signal is given whereas the RF signal towards the antenna is derived. This needs a different setup of the partial DAE for the up- and downlink. Both scenarios have been realized and tested. A simulation result for the c9linmix mixer example from the test suite is depicted in Fig. 11 and Fig. 12 respectively. Fig. 11 shows the PDE solution of the mixer output for a downlink scenario. Fig. 12 illustrates the univariate time domain solution obtained along a characteristic curve of the PDE.

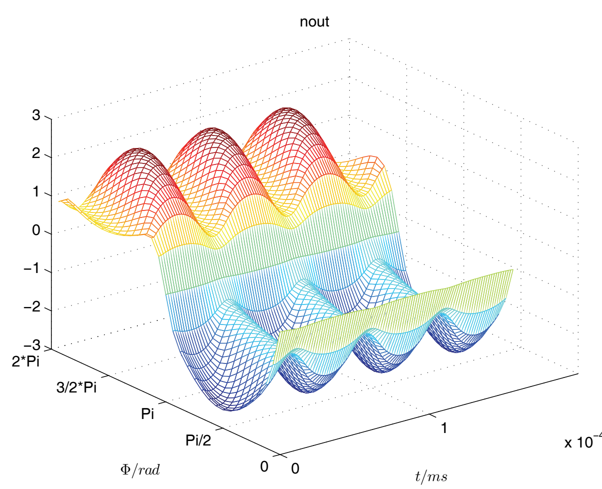

Figure 11: Bivariate solution of the PDE (2) of the mixer test example.

3.5.2 Sweep Following method

An alternative method for warped multirate PDEs, Optimal Sweep Following, has been further developed between NXP and WUP. It exploits also the multitime formulation (2) but adds an additional time splitting on top of it in an adaptive way. This time splitting can be done in an optimal and dynamic way by which one efficiently determines the solution along a kind of a telephone cord in two directions in which along one direction the main activity is concentrated. This is the fast direction. In contrast in the other direction the activity can be influenced by our choice of the local frequency function. The optimal choice yields a relatively low activity

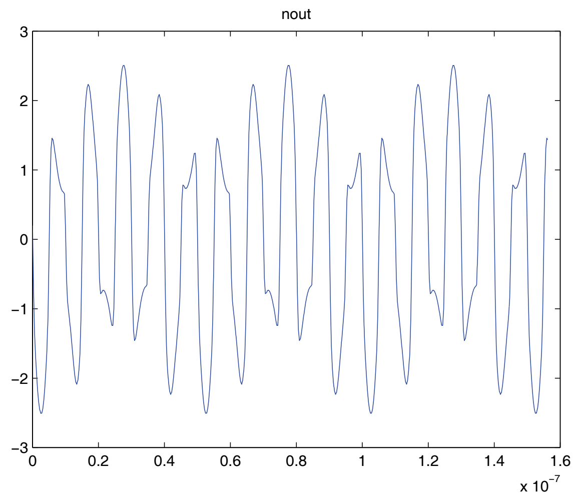

Figure 12: Reconstructed univariate solution of the underlying ordinary DAE (1) of the mixer test example.

and allows here to take large stepsizes. Inappropriate choices can make the activity arbitrarily large. The time splitting dynamically improves a time splitting that was initiated by a user. The windings around the cord represent the fastly varying component. In the direction along the cord one can follow the slowly varying component. The method applies both to amplitude and to frequency modulated problems. The time splitting idea can be included in the other multirate methods (T1.2 and T1.3) as well. The reconstruction of the solution of the original univariate problem is done similarly as for the wavelet based multirate methods. The envelope of the univariate solution can also be efficiently reconstructed.

The method has been tested both on driven and on autonomous (i.e. free running) oscillators. Fig. 13 shows the transient solutions of the univariate problem (1) of an autonomous Colpitts oscillator. Fig. 14 depicts the bivariate solution of one of the components of the solution of the multitime formulation (2). The reconstruction of the univariate solution obtained from this bivariate solution is shown in Fig. 15. This compares well with the corresponding curve in Fig. 13. Fig. 15 also shows the upper and lower envelopes, which are efficiently obtained.

4 Conclusion

Task T1.1 was called "Wavelet-based algorithms for mixed analogue/digital simulations". Standard methods approximate the signals by a truncated Taylor series or a rational polynomial. This approximation is however not well suited for the signals occurring in mixed signal simulations. Consequently, Task T1.1 addressed an alternative by employing a wavelet basis.

A new adaptive wavelet-based method has been developed and a prototype implemented in the IFX in-house circuit simulator TITAN. Non-uniform spline wavelets are used for the adaption of a spline knot grid, while the Newton solver is based on the more efficient B-spline representation. A fast wavelet transform is used for the fast change between both representations.

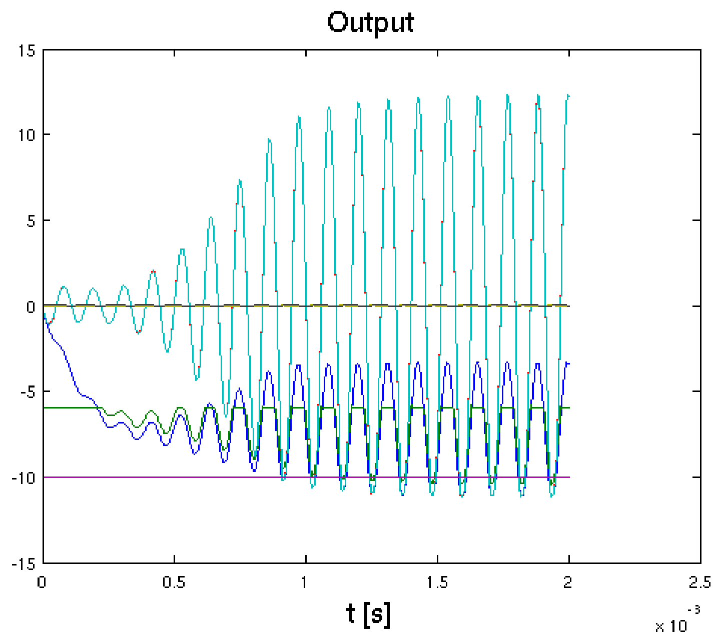

Figure 13: Transient solutions of the univariate problem (1) of a Colpitts oscillator.

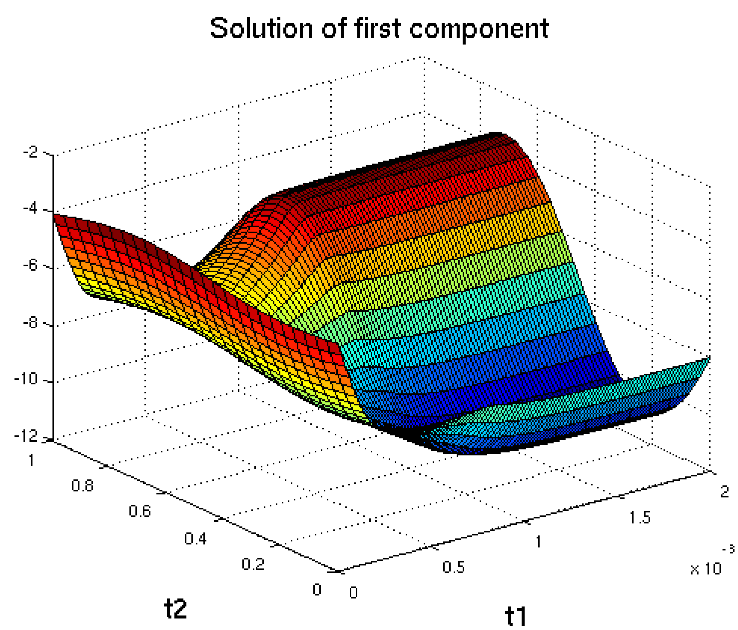

Figure 14: One of the bivariate solutions of the multitime formulation (2) of the Colpitts oscillator using the Sweep Following method.

The method can simulate both general circuits as well as a special class of periodic circuits, both autonomous and non-autonomous oscillators. The validation tests indicate that the wavelet-based algorithms can reach and surpass the accuracy of traditional time stepping methods, e.g., classical transient analysis. The tests also show that an improvement in performance is needed

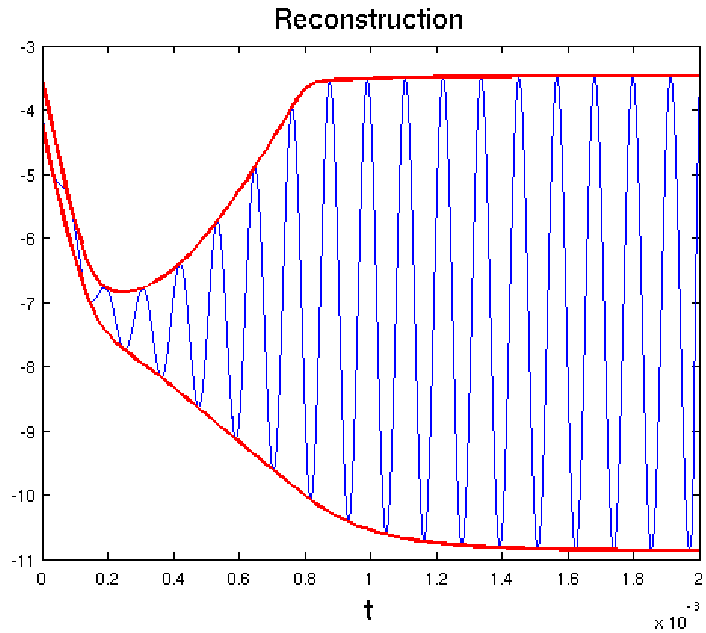

Figure 15: Reconstruction of the univariate solution out of the bivariate solution in Fig. 14. The upper and lower envelopes were calculated as well.

before the wavelet-based methods can be fully exploited in an industrial environment.

Within Task T1.2 "Multirate methods based on wavelet approaches" broadband signals such as CDMA (Code Division Multiple Access[1]) or OFDM (Orthogonal Frequency-Division Multiplexing[2]) signals are represented in a much more efficient way than traditional techniques. The multirate problem is reformulated by a partial differential equation (PDE). A wavelet basis allows for the construction of adaptive grids in the multivariate domain of dependence.

Starting from the general wavelet technique a prototype of the multirate wavelet method based on periodic wavelets was also developed and implemented in TITAN. Currently this implementation is available only for non-autonomous circuits. Validation tests showed that the accuracy requirements of the method are fully met, but, similar to the basic method, there is still a lot of options for the performance optimization.

Considering that the existing prototype of the wavelet technique is already able to completely fulfill the accuracy requirements for different classes of circuits under simulation and that there exists high potential for further performance optimization, we are convinced that wavelet analysis has a high potential for industrial usage in the future. Therefore partner IFX intends to continue with the wavelet analysis optimization beyond the Project duration.

Task T1.3 "Multirate Envelope Methods" addressed the general problem of simulating circuits driven by multirate signals originating from demands of state-of-the art modulation techniques as discussed above. This task used the same (PDE) formulation as Task T1.2. The multirate envelope method based on embedding the system of ordinary DAEs into partial DAEs [6] is the only technique with a sound mathematical basis. It exhibits a general mathematical framework

[1] http://en.wikipedia.org/wiki/Code_division_multiple_access

[2] http://en.wikipedia.org/wiki/Orthogonal_frequency-division_multiplexing

which can be adapted to different classes of RF circuits. It has been implemented and successfully verified for a large class of circuits, namely oscillators and mixers. For the last ones a different setup of the partial DAE for the up- and downlink scenario was needed. In a future project this technique shall be extended to other classes of RF circuits, i. e. PLLs. A first step in this direction has been done by successfully proving the reliability of the method for VCOs. It is still not clear however if this method can be employed for frequency divider circuits which are main building blocks of PLLs. However the authors are optimistic to solve this problem in future projects. The method has been implemented in the in-house simulator LinzFrame of FHO.

An alternative method, Optimal Sweep Following, has been developed in cooperation between NXP and WUP. The Sweep Following method represents a particular strategy for the determination of a suitable time varying phase shift function α or a local frequency function α' that allows an optimal time stepping for warped multirate PDEs. The optimal time splitting dynamically determines optimal time directions in which along one direction the main activity is concentrated. This is the fast direction. This activity is problem-dependent and can be low or high, depending on the circuit. In contrast in the other direction the activity can be influenced by our choice of the local frequency function. The optimal choice yields a relatively low activity and allows here to take large stepsizes. Inappropriate choices can make the activity arbitrarily large. This idea can be included in the other multirate methods (T1.2 and T1.3) as well.
The time splitting dynamically improves a time splitting that was initiated by a user. The method applies both to amplitude and to frequency modulated problems. The reconstruction of the solution of the original univariate problem is done similarly as for the wavelet based multirate methods. The envelope of the univariate solution can also be efficiently reconstructed.

The method has been tested successfully on driven and autonomous oscillators, including Colpitts and Pierce oscillators. This did result in several improvements of the method. The method is best suitable to non-autonomous problems. Because of its strategy the autonomous problems were a real test for robustness. This has led to an adapted criterion (balancing stepsize against condition number) as well as to scaling approaches. Both generalizations deserve some further research.
A basic implementation is available to all partners.

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