Entrainment phenomena in nonlinear oscillations

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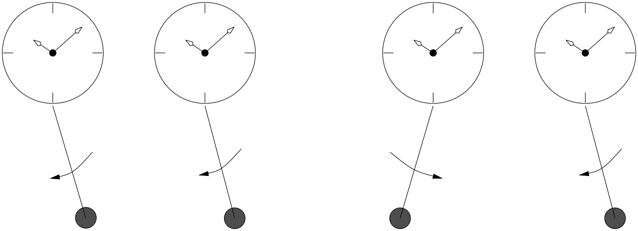
Outline

1. Entrainment phenomena
2. Basic concepts
3. Theory of entrainment
4. Simulations
5. Circuit designs

Entrainment Phenomena · Christiaan Huygens 1629 - 1695

Christiaan Huygens' experiment

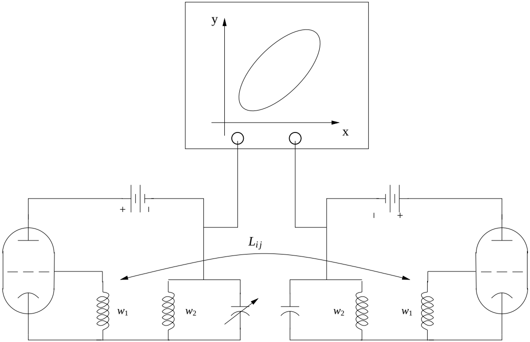


Entrainment Phenomena · A Universal Phenomenon

- Mode-locked lasers for the suppression of phase noise
- Lock range of a PLL; lock range $>$ capture range
- Circadian rhythm, regulation of the light-dark cycle
- Cardiac rhythm, pacemaker
- Collective flashing of the southeast Asian fireflies
- Social events, rock concerts, political party meetings
- Circuit designs; Appletion, Melville et al., Gierkink et al.

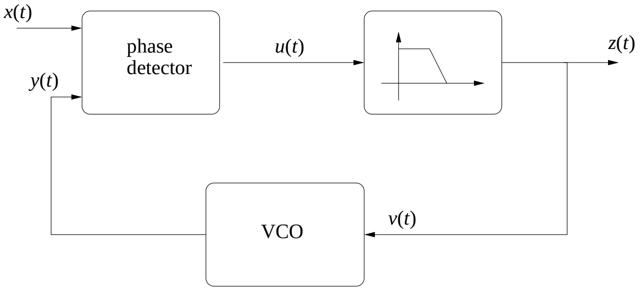
Entrainment in Circuit's - Appleton's experiment

Sir Edward Appleton's experiment of mutual entrainment



Phase Entrainment - The PLL

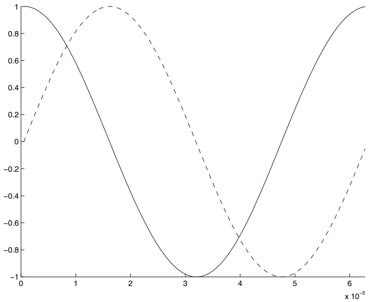
Phase-Lock-Loop as example of phase entrainment



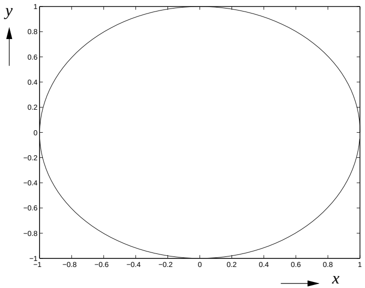
Entrainment Phenomena · Basic concepts

Van der Pol Oscillator

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} - \omega_0 \begin{pmatrix} v_1(1 - v_2\, y^2) & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$



(a) $x(t)$ and $y(t)$



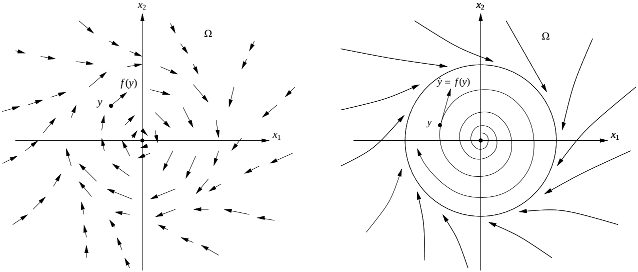
(b) y versus x

Entrainment Phenomena · Basic concepts

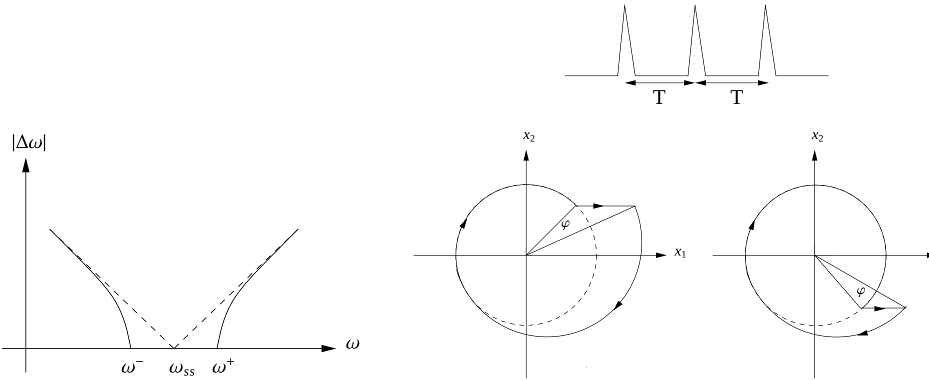
System of autonomous ODEs

$$\dot{x} = f(x)$$

with periodic limit cycle $x_{ss}(t) = x_{ss}(t + T)$
Representation in phase space



Entrainment Phenomena · Impulse train



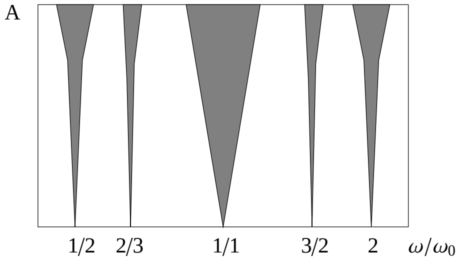
(a) Detuning (b) Orbital and phase deviations

ω^-, ω^+ bifurcation points;

define normalized locking range $\delta := \frac{\omega^+ - \omega^-}{\omega_{ss}}$

Entrainment Phenomena · Arnold tongue

Region of Entrainment as function of signal amplitude & detuning

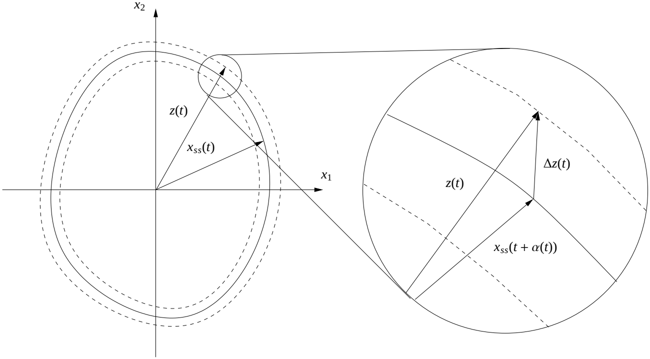


Theory of entrainment · System of ODEs

Consider driven autonomous system

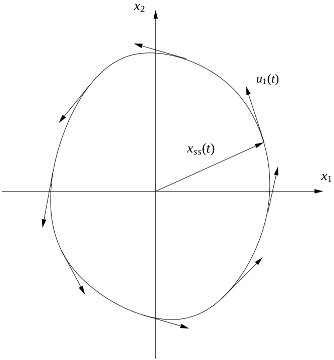
$$\dot{x} = f(x) + b(t)$$

where $b(t) = b(t + \hat{T})$ with $\hat{T}$-periodic solution $z(t)$



Theory of Entrainment · Floquet-Theory

The Floquet eigenvector $u_1(t)$ is tangent to the limit cycle

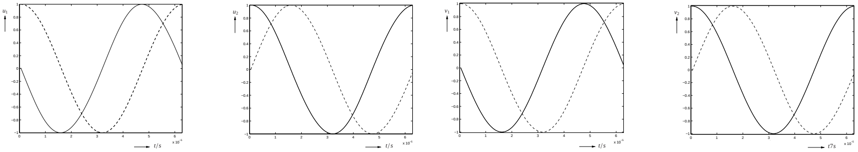


and causes stationary deviation

Theory of Entrainment · Floquet-Theory

Van der Pol Oscillator

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} - \omega_0 \begin{pmatrix} v_1(1 - v_2\, y^2) & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A \sin(\Omega\, t) \\ 0 \end{pmatrix}, \quad \Omega = \frac{2\,\pi}{\hat{T}}$$



(a) $u_1(t)$ (b) $u_2(t)$ (c) $v_1(t)$ (d) $v_2(t)$

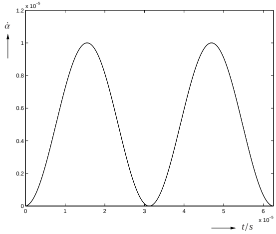
Theory of Entrainment · Simulations

Maximizing and Minimizing of

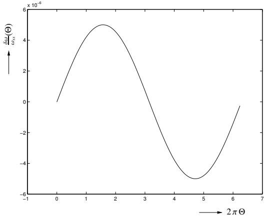
$$\alpha_0(\Theta) = \frac{1}{\hat{T}} \int_0^{\hat{T}} v_1^T(t + \alpha_\Theta(t))\, b(t + \Theta\, \hat{T})\, \mathrm{d}t, \quad 0 \le \Theta < 1$$

Van der Pol oscillator:

$$\delta \approx \alpha_{0_{\mathrm{max}}} - \alpha_{0_{\mathrm{min}}} = 1{,}00031 \cdot 10^{-5}$$



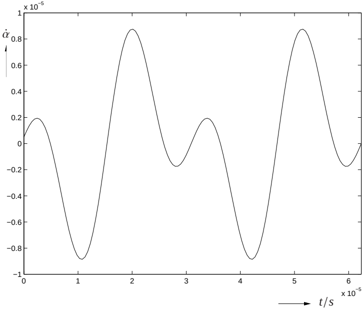
(a) $\frac{\mathrm{d}\alpha_{\mathbf{max}}(t)}{\mathrm{d}t}$



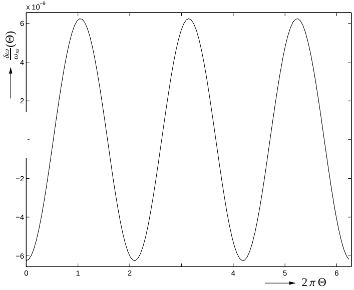
(b) $\frac{\delta\omega}{\omega_{ss}}(\Theta)$

Superharmonic Entrainment · Simulations

Superharmonic entrainment of the Van der Pol oscillator



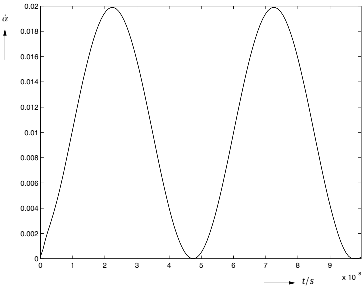
(a) $\frac{\mathrm{d}\alpha_{\mathbf{max}}(t)}{\mathrm{d}t}$



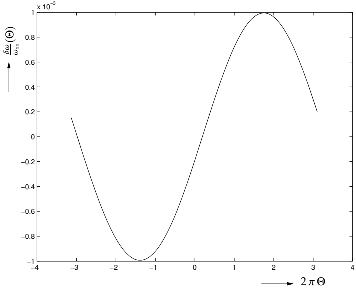
(b) $\frac{\delta\omega}{\omega_{ss}}(\Theta)$

Superharmonic Entrainment · Simulations

$$\delta = \alpha_{0_{\max}} - \alpha_{0_{\min}} = 1{,}9668 \cdot 10^{-3}$$



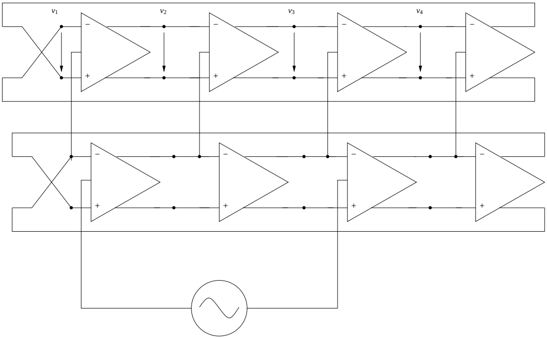
(a) $\frac{\mathrm{d}\alpha_{\mathbf{max}}(t)}{\mathrm{d}t}$



(b) $\frac{\delta\omega}{\omega_{ss}}(\Theta)$

Entrainment · Circuit designs · Quadrature oscillator

Quadrature circuit by Robert Melville, Peter Kinget et al.,
Lucent Technologies, Bell Laboratories



Entrainment · Summary

- Theory of entrainment given by a first order perturbation along the limit cycle.
- Theory based on Floquet's theory for linear periodically time-varying ODEs.
- Theory can be expanded to DAEs; Winkler, Lamour, März, Winkler and Demir.
- Future work on first and second order perturbations.