Estimation of Initial Conditions for Harmonic Balance Oscillator Analysis of Free-Running Oscillators

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Outline

Introduction to Oscillator Analysis

Start-up of an Oscillator (TRANsient simulation)
  Colpitts Oscillator
  VCOBi Oscillator

HB Oscillator Analysis in the APLAC Simulator

Algorithms for Estimating the Initial Conditions
  FFT-based Method
  Zero-Crossings Method

Simulation Results

Conclusions and Further Improvements

New Newton method using generalized eigenvalues

Conclusions new Newton method
Introduction to Oscillator Analysis

- Oscillator is an autonomous circuit, no periodic excitation
  - Only DC bias sources connected to the circuit
- Transient analysis of an oscillator
  - Oscillation starts by itself due to noise or unstability
  - Long start-up time ⇒ long simulation time
  - Suitable for highly nonlinear oscillators
- HB analysis of an oscillator
  - May converge to DC solution (no periodic excitation!)
  - Need to either modify HB equation to be solved, or apply artificial excitation
  - Oscillation frequency, i.e, fundamental HB frequency, is unknown
- Why use HB instead of transient analysis?
  - HB is needed for (phase) noise simulations
  - Frequency dependent linear devices (S-parameters, ...)

Estimation of Initial Conditions for Harmonic Balance...
Colpitts: $f_{osc}=10.007$ MHz

Steady-state:
40 periods of $f_{osc}$

Colpitts Oscillator: TRANsient start-up

Estimation of Initial Conditions for Harmonic Balance...
Start-up of an Oscillator (TRANsient simulation)

VCOBi Oscillator

VCOBi: fosc=1.23 GHz

Steady-state:
7 periods of fosc

Estimation of Initial Conditions for Harmonic Balance...
HB Oscillator analysis is done using optimization; variables are oscillation frequency, fosc, and oscillation amplitude, vosc.
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Artificial excitation $\text{OscAProbe}$: voltage source (in series with a non-zero resistor: saves DAE-index!) is connected to the circuit. Optimization goal is to have zero current through the probe element.
HB Oscillator analysis is done using optimization; variables are oscillation frequency, \( f_{osc} \), and oscillation amplitude, \( v_{osc} \).

Artificial excitation \( \text{OscAProbe} \): voltage source (in series with a non-zero resistor: saves DAE-index!) is connected to the circuit. Optimization goal is to have zero current through the probe element.

Oscillator analysis algorithm

1) Change optimization variables \( f_{osc}, v_{osc} \)
2) HB analysis (incl. nonlinear iteration)
3) Compute optimization goal
4) Terminate optimization if in specs, otherwise goto 1)
Two algorithms for finding better initial estimate for the oscillator analysis are presented

- FFT-based Method (search for the most dominant spectral line)
- Zero-Crossings Method (estimate frequency based on zero crossings) – Related to Poincaré Map method

Both of these methods use the data from the transient simulation, length of the period is $1/f_{osc_init}$ (initial value of $f_{osc}$):

- transient analysis is first run up to five periods;
- continue transient for one period, and store the probe element voltage waveform

Transient start-up can be accelerated by injecting a short $\cos()$-like pulse (length less than one period)
FFT-based Method

a) First, do FFT for the transient voltage waveform
b) Search for the most dominant spectral component at non-zero frequency, and
c) a parabola (second order polynomial) is fitted to the most dominant and neighboring spikes at both sides, and
d) the maximum of the parabola is computed
e) \( \Rightarrow \) this method can be used to find both frequency and oscillation amplitude
Zero-Crossings Method

a) First, the average value or half of the maximum swing is removed from the collected time-domain data, and

b) zero-crossings are searched from the data

c) Accuracy can be improved by applying linear or spline interpolation, i.e., interpolation is done for 1-2 data points on both sides of the zero-crossing location

d) Oscillation amplitude can be estimated from the maximum swing
Comparison of Implemented Methods
VCOBi oscillator: \( f_{osc} = 1.23\) GHz, \( v_{osc} = 1.43\) V, \( t_0 = 5/f_{oscinit} \)

<table>
<thead>
<tr>
<th>method</th>
<th>HBITER</th>
<th>CPU[s]</th>
<th>estimated ( f_{osc} )[GHz]</th>
<th>estimated ( v_{osc} )[V]</th>
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<tbody>
<tr>
<td>old</td>
<td>1116</td>
<td>3.1</td>
<td>1.0</td>
<td>1.0</td>
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<td>FFT/1</td>
<td>58</td>
<td>0.1</td>
<td>1.11</td>
<td>0.945</td>
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<td>FFT/2</td>
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<td>4.0</td>
<td>1.13</td>
<td>0.948</td>
</tr>
<tr>
<td>ZeroC/1</td>
<td>23</td>
<td>0.06</td>
<td>1.23</td>
<td>1.36</td>
</tr>
<tr>
<td>ZeroC/2</td>
<td>31</td>
<td>0.08</td>
<td>1.22</td>
<td>1.41</td>
</tr>
</tbody>
</table>

/1 set \( f_{osc} \) and \( v_{osc} \), spectral voltages from transient simulation
/2 use initial pulse and /1
Initial values:
\(f_{osc}=3.0\;\text{GHz},\;v_{osc}=1.0\;\text{V}\)

<table>
<thead>
<tr>
<th>method</th>
<th>HBITER</th>
<th>CPU[s]</th>
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</thead>
<tbody>
<tr>
<td>old</td>
<td>2259</td>
<td>110.6</td>
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<tr>
<td>TAHB</td>
<td>48</td>
<td>2.7</td>
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<tr>
<td>FFT</td>
<td>47</td>
<td>6.2</td>
</tr>
<tr>
<td>ZeroC</td>
<td>31</td>
<td>1.4</td>
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</table>

Estimated \(f_{osc}\) and \(v_{osc}\):

<table>
<thead>
<tr>
<th>method</th>
<th>(f_{osc})</th>
<th>(v_{osc})</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT</td>
<td>2.7 GHz</td>
<td>1.09 V</td>
</tr>
<tr>
<td>ZeroC</td>
<td>3.1 GHz</td>
<td>1.17 V</td>
</tr>
</tbody>
</table>
Conclusions and Further Improvements

- Usage of these new methods improves reliability and robustness of HB oscillator analysis: User does not need to supply accurate initial values
- Zero-Crossing methods seems to be better than FFT method (Q: Multiple crossings?)
- Optimization algorithms are quite sensitive to the formulation of the optimization goal and simulation set-up. Better initial estimate does not always result in faster convergence in optimization and/or HB analysis

- TODO: tune optimization algorithm and/or parameters more suitable for oscillator optimization
- TODO: add more adaptivity, currently initial transient is optionally continued until estimate of vosc is within user-specified limits
New Newton method for PSS

DAE-system of equations on $[0, T]$

$$\frac{dq(x)}{dt} + j(x) = 0 \in \mathbb{R}^N,$$

where $x = x(t), x(0) = x(T), q$ and $j$ are known functions of $x$.

Transforming to $[0 1] \rightarrow f = 1/T$ explicit unknown

$$f \frac{dq(x)}{dt} + j(x) = 0.$$

One usually requires $p^T x - c = 0 \rightarrow$ which $p$, which $c$?

Usually left to the user ...

Simpson’s Rule un $[t_{i-1}, t_{i+1}]$

$$F(X^k) = \frac{x_0 - x_N}{2\Delta t} + \frac{(j_0 + 4j_1 + j_2)}{6} \quad \cdots \quad \frac{x_N - x_{N-1}}{2\Delta t} + \frac{(j_{N-1} + 4j_N + j_1)}{6}$$

Newton:

$$M^k \begin{bmatrix} \Delta X^k \\ \Delta f^k \end{bmatrix} = - \begin{bmatrix} F(X^k, f^k) \\ p^T X^k - c \end{bmatrix}$$

$$M^k = \begin{bmatrix} Y^k & b^k \\ p^T & \delta \end{bmatrix}$$

$$Y^k = \frac{\partial F}{\partial x} = f \cdot c + g, \quad b^k = \frac{\partial F}{\partial f}$$

- how dynamically improve $p, c$?
Use of generalized eigenvalues

• Observ. 1: $Y$ nearly singular in the limit $\implies p$ and $b$ such that $M$ non-singular. $c$ must be in a proper range.
• Observ. 2: $x'_{\text{PSS}}(t)$ satisfies linearized equations.

Generalized eigenvalues for matrix pencils:
• DPA [Dominant Pole Algorithm]
• RQI [Raleigh Quotient Iteration]

Eigen triples $[v, w, \lambda]$ for $\lambda$ closest to 1

$$[\lambda f \times C + G]v = 0, w^T[\lambda f \times C + G] = 0$$

Biorthogonality relation $w^T C v = 1 \implies$ new gauge equation

$$w^T \left|_{x=x^k} \right. \cdot C \cdot \left. \frac{dX}{dt} \right|_{x=x^k} - 1 = 0.$$ 

$$dX/dt = DX$$ [spectral differentiation]
Benchmark problem

\[
\frac{\partial y}{\partial t} = z + \varepsilon h(y, z) y,
\]
\[
\frac{\partial z}{\partial t} = -y + \varepsilon h(y, z) z.
\]

\( \varepsilon = 0.1 \)
\( h(y, z) = 1 - \sqrt{y^2 + z^2} \)

Initial conditions:
\( T_0 = 1.1 \times (2\pi) \)
\( y_0(t) = 1.5 \sin(t + \pi/4) \)
\( z_0(t) = \cos(t) \)

100 time points

Compare Gauge equations:
- Using \( c \) outside range of \( x(t) \)
- Using better \( c \)
- Using generalized eigenvalues
Conclusions new Newton method

- Traditional methods require knowledge by the user:
  - Location of the oscillator solution \((p)\)
  - Range of values of the oscillator solution \((c)\)
- Generalized eigenvalue methods dynamically provide a new gauge equation
- Spectral differentiation of \(x(t)\) is natural in Harmonic Balance
- New approach looks robust
- TODO: tests needed on larger oscillator circuits

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