Wavelet algorithm for circuit simulation

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Wavelet expansions

- Representation of $f : [a, b] \to \mathbb{R}$ as

$$f = \sum_{k \in \mathcal{I}} c_k \, \phi_k + \sum_{j=0}^{\infty} \sum_{k \in \Lambda_j} d_{jk} \, \psi_{jk}.$$

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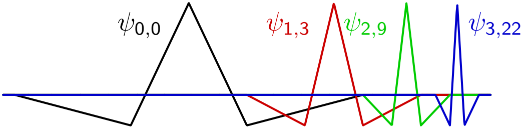
- Details of increasing resolution j in terms of wavelets ψ_{jk}



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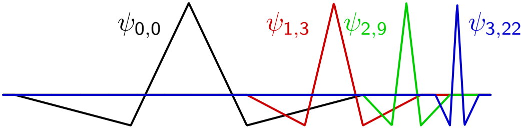

- Infinite representation $\rightsquigarrow$ finite approximation needed

Wavelet approximation

Thresholding: $$f \approx f_\varepsilon := \sum_{k \in \mathcal{I}} c_k \, \phi_k + \sum_{(j,k):\ |d_{jk}| > \varepsilon} d_{jk} \, \psi_{jk}$$

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e.g., $$\sum_{k\in\mathcal{I}_J} f(t_{J,k})\,\phi_{J,k} = \sum_{k\in\mathcal{I}} c_k\,\phi_k + \sum_{j=0}^{J}\sum_{k\in\Lambda_j} d_{jk}\,\psi_{jk}.$$

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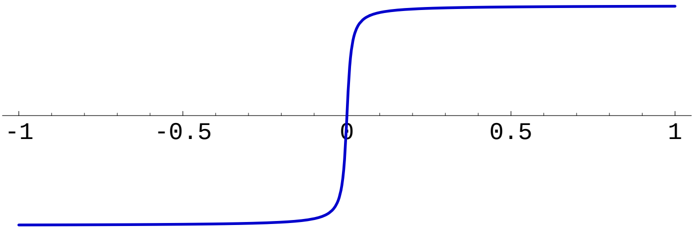
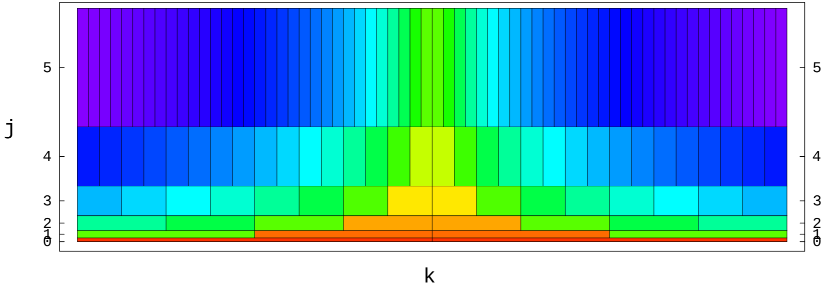
$\{f(t_{J,k})\} \to \{c_k\}, \{d_{jk}\}$ by fast wavelet transform.

Example of wavelet expansion





Magnitude of wavelet coefficients of function f

Important properties of Wavelets

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$$\rightsquigarrow \quad \left|d_j^k\right| \le C\, 2^{-j\alpha} \left\| f|_{\operatorname{supp} \psi_{jk}} \right\|_{W_p^\alpha}, \quad \alpha < m$$

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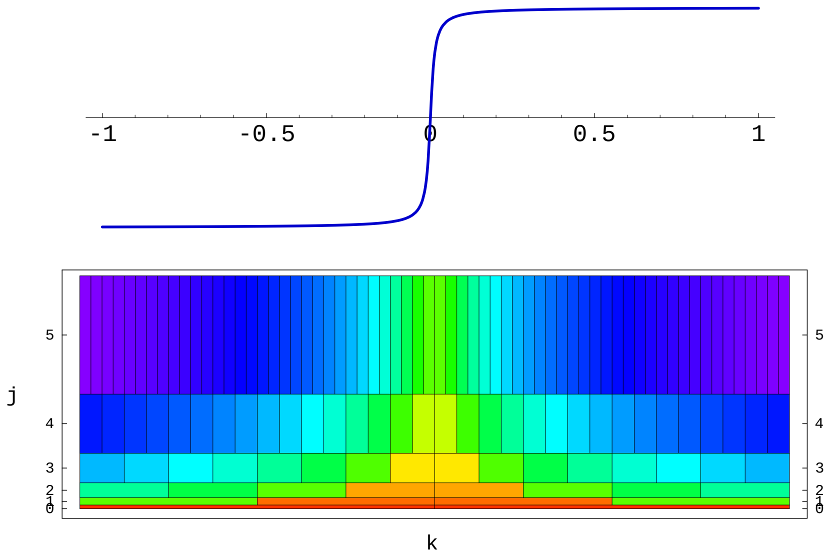

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Adaptive Wavelet Solver

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$$\frac{d}{dt}q\big(x(t)\big) + g\big(x(t)\big) = s(t)$$

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  - adaptive representation $\rightsquigarrow$ minimal size of problem

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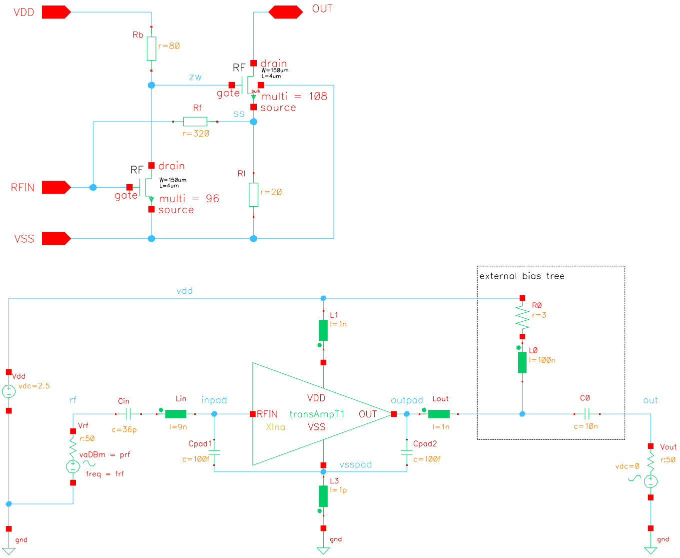
  - Amplifier
  - Mixer
  - Oscillator

Example — Amplifier



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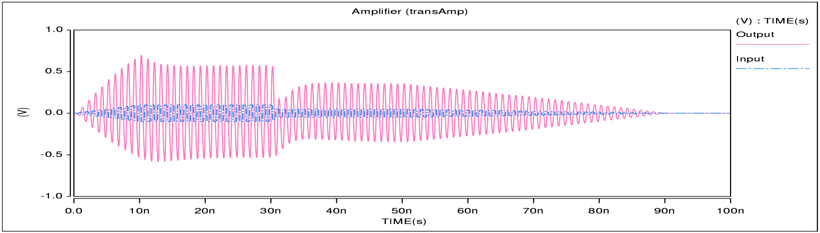
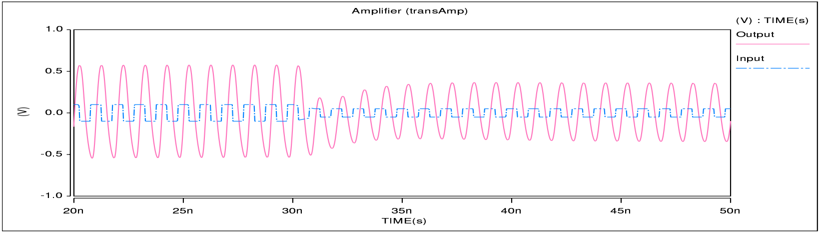
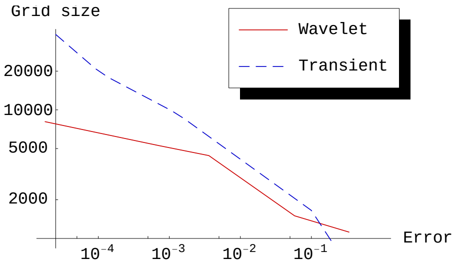


Input and output signal

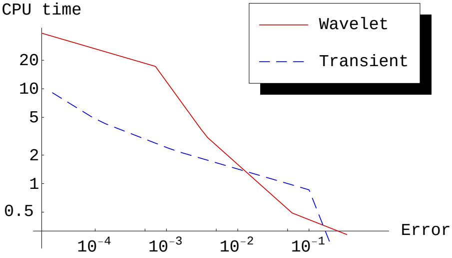


Detail

Example — Amplifier

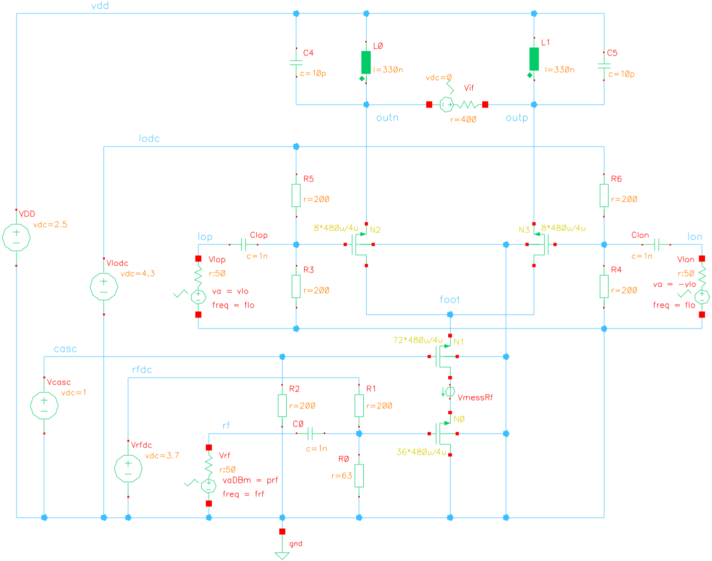


Grid size versus error



CPU time versus error

Example — Mixer



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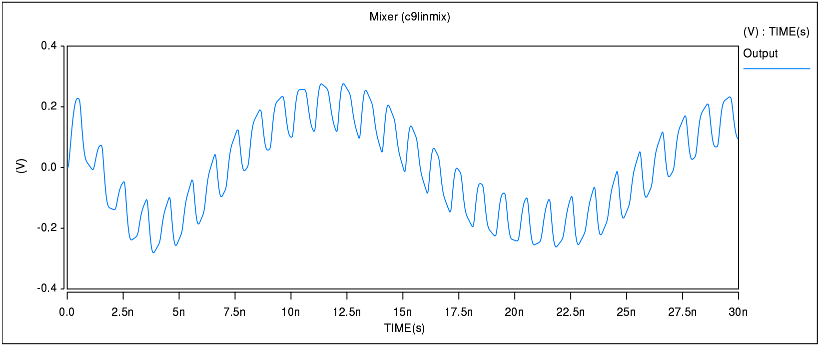

Output for $f_{rf} = 950\text{MHz}$ and $f_{lo} = 1\text{GHz}$

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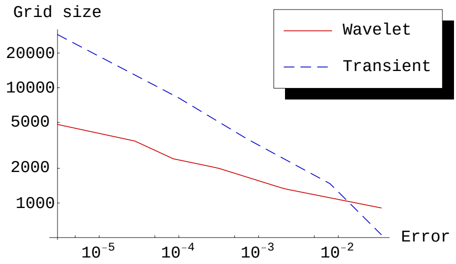

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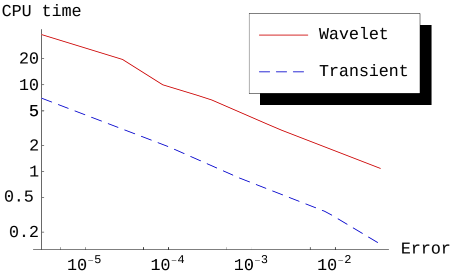

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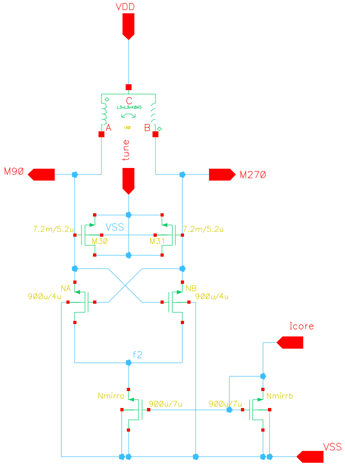

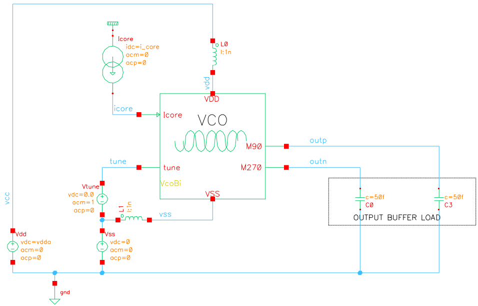

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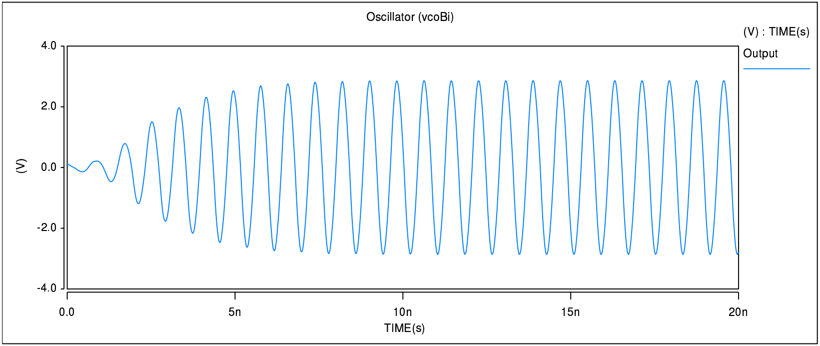

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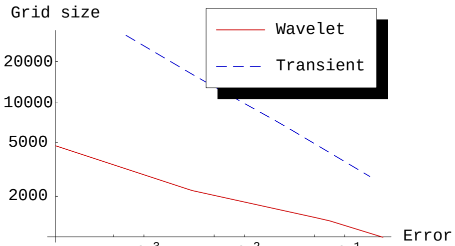

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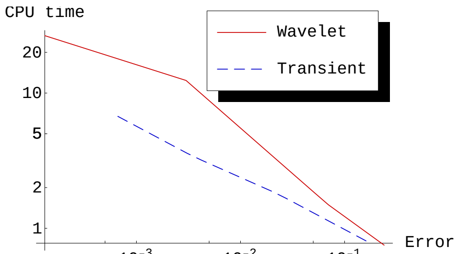

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- Further optimization needed before released to end-user

Thank you

Thank you for your attention!

Questions?