A Mixed Time Frequency Algorithm for Circuit Simulations

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Introduction

- At the high data rates requires huge signal bandwidths and high center frequency of several GHz led CAD tools to their limits.
- A novel method has been developed to circumvent Nyquist rate problem.
- The method is based on reformulating the ordinary DAE to a system of PDEs, also known as multirate PDEs (MPDEs).
- Formulation of PDE depends on the circuit class under investigation.
- The formulation of PDE also depends on number of fundamental tones or frequencies, therefore autonomous and non-autonomous circuits can be treated.
- Autonomous circuits comprise mainly oscillators, the frequency of oscillations is not known a-priori.
- The PDE formulation differs significantly from autonomous to non-autonomous case.

Cont...

Consider the system of ordinary DAEs

$$\frac{d}{dt}\mathbf{q}(\mathbf{x})(t) = \mathbf{f}(\mathbf{b}(t), \mathbf{x}(t)) \quad (1)$$

$$\mathbf{x}_0 = \mathbf{x}(t_0) \quad (2)$$

where, $\mathbf{x} : \mathbb{R} \to \mathbb{R}^N$, $\mathbf{b} : \mathbb{R} \to \mathbb{R}^N$ and $\mathbf{f} : \mathbb{R}^N \times \mathbb{R}^N \to \mathbb{R}^N$
Introduce $\hat{\mathbf{x}} : \mathbb{R}^m \to \mathbb{R}^N$ for the state variables and $\hat{\mathbf{b}} : \mathbb{R}^m \to \mathbb{R}^N$ of the input signals.
For simplicity, we consider the function $\mathbf{f}$, $\mathbf{b}$, $\hat{\mathbf{b}} \in \mathcal{C}^0$ and $\mathbf{q}$, $\mathbf{x}$, $\hat{\mathbf{x}} \in \mathcal{C}^1$. In 1996, Brachtendorf et al. (Numerical steady state analysis of circuits driven by multi-tone signals, published in Elect. Eng.) have introduced the corresponding multirate partial differential algebraic equation (MPDAE)

Formulation

$$\left(\frac{\partial}{\partial\tau} + \frac{\partial(\tau\omega_1(\tau))}{\partial\tau}\frac{\partial}{\partial t_1} + \ldots + \frac{\partial(\tau\omega_{m-1}(\tau))}{\partial\tau}\frac{\partial}{\partial t_{m-1}}\right)\mathbf{q}(\hat{\mathbf{x}})$$
$$= \mathbf{f}(\hat{\mathbf{b}}(\tau, t_1, \ldots, t_{m-1}), \hat{\mathbf{x}}(\tau, t_1, \ldots, t_{m-1})).$$

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Important Result

A given solution $\hat{\mathbf{x}}$ of the MPDAE (3) coincides with a solution $\mathbf{x}$ of DAE (1) along the curve *i.e.* characteristic curve

$$\mathbf{x}(t) = \hat{\mathbf{x}}(t, \omega_1 t, \ldots, \omega_{m-1} t). \tag{3}$$

In Radio Frequency (RF) applications, many system comprises exactly two different time scales :

MPDAE

$$\left(\frac{\partial}{\partial\tau} + \frac{\partial(\omega(\tau)\tau)}{\partial\tau}\frac{\partial}{\partial t_1}\right)\mathbf{q}(\hat{\mathbf{x}}(\tau, t_1)) = \mathbf{f}(\hat{\mathbf{b}}(\tau, t_1), \hat{\mathbf{x}}(\tau, t_1)) \tag{4}$$

Cont...

$$\begin{aligned} \hat{\mathbf{x}}(0, t_1) &= \mathbf{h}(t_1) \ \ \forall t_1 \in \mathbb{R}, \\ \hat{\mathbf{x}}(\tau, t_1) &= \hat{\mathbf{x}}(\tau, t_1 + T) \ \ \forall \tau, t_1 \in \mathbb{R}, \end{aligned} \tag{5}$$

$$\hat{\mathbf{x}}(\tau, t_1) \approx \sum_{k=-K}^{K} \mathbf{X_k}(\tau) \exp\left(i\omega(\tau) k t_1\right) \tag{6}$$

The Fourier coefficient $\mathbf{X}_k : \mathbb{R} \to \mathbb{C}^N$ with $k = -K, \ldots, K$.

Cont...

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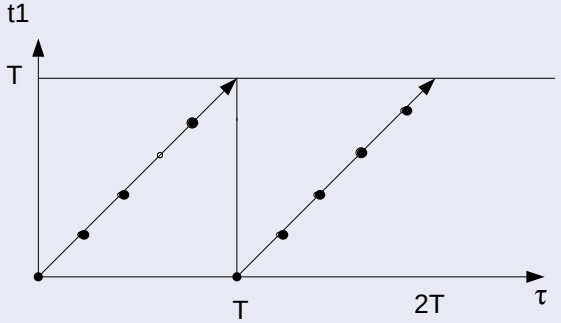
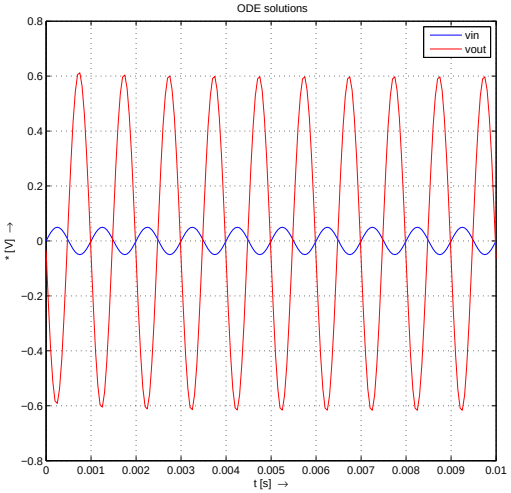
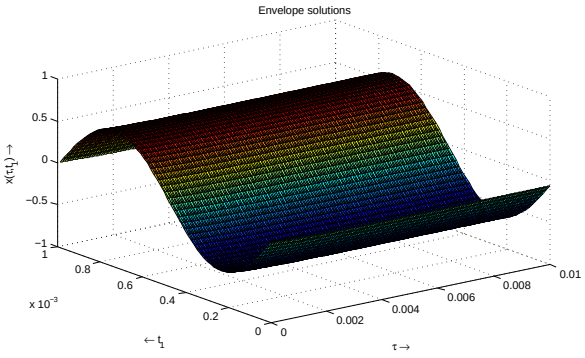
Initial values estimation, Stephanie Knorr, Wavelet-Based Simulation of MPDAS in RF Applications Ph.D. Dissertation, Univ. of Wup, 2007



Amplifier Circuit

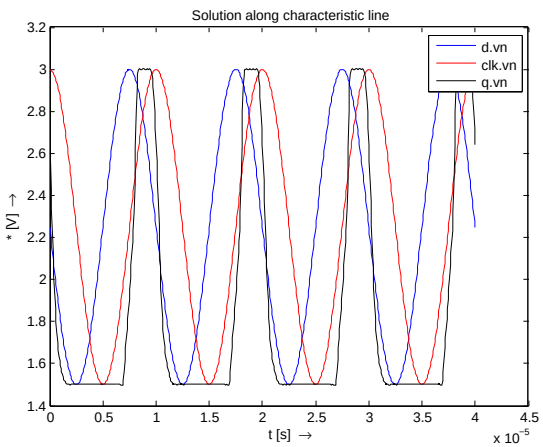




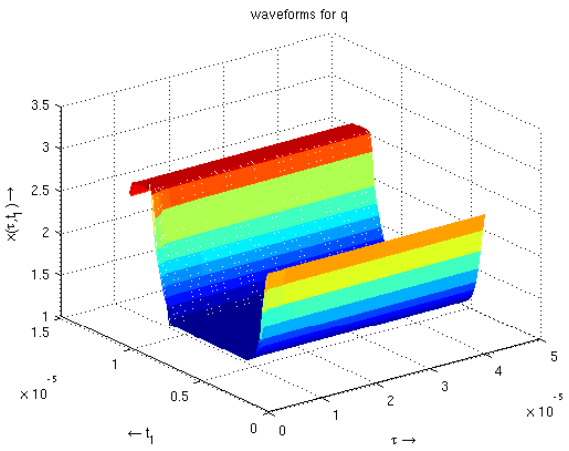
(a) (b)

Figure : (a) Solutions along characteristic curve, (b) Waveforms for output voltage

Differential Flip-Flop Circuit



(a)



(b)

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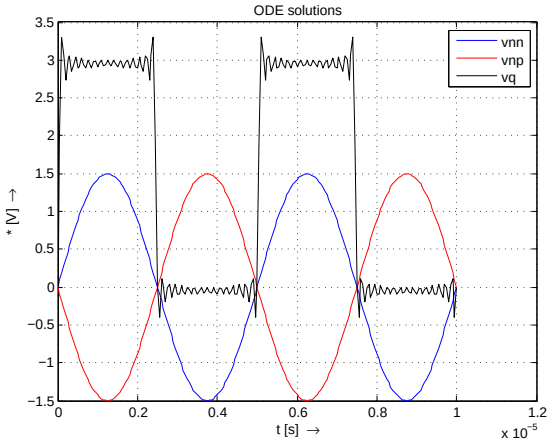
Differential to Single Circuit



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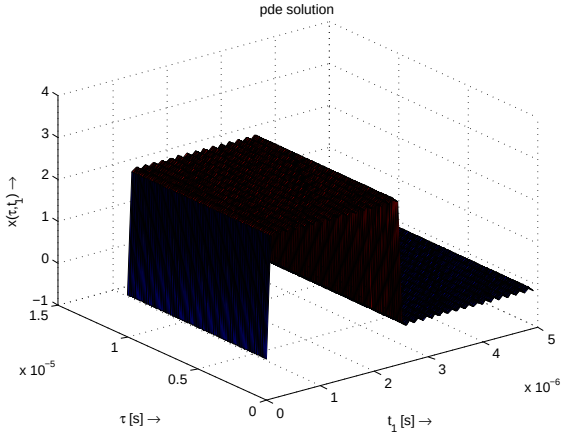

(b)

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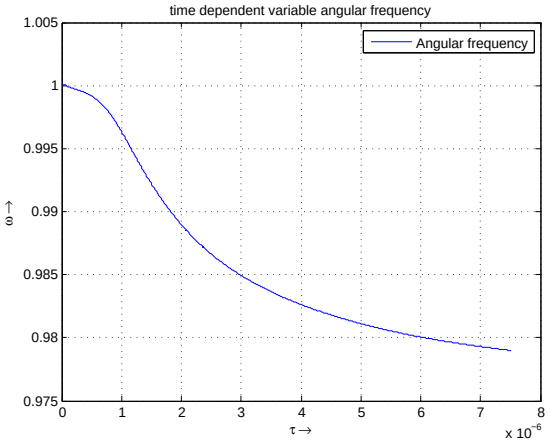
Colpitts Oscillators



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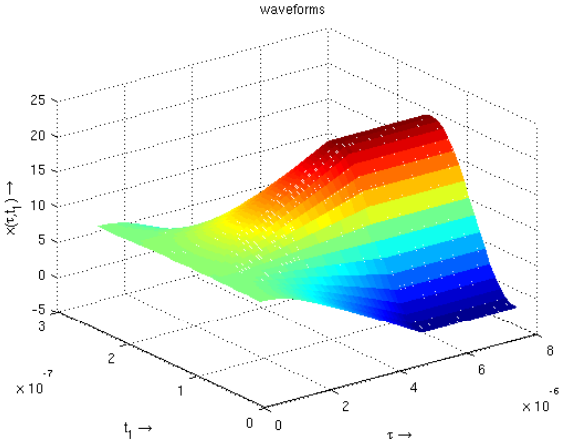

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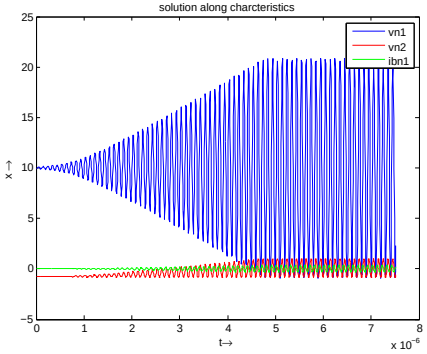
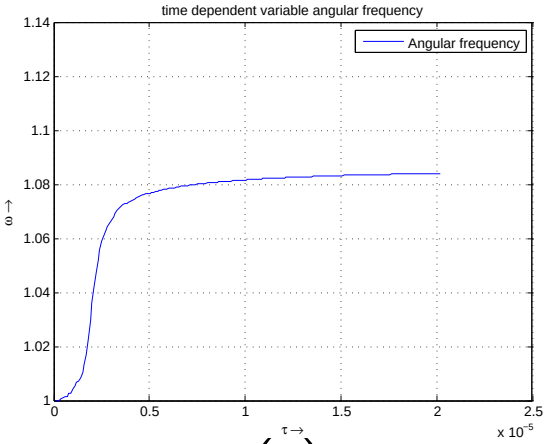


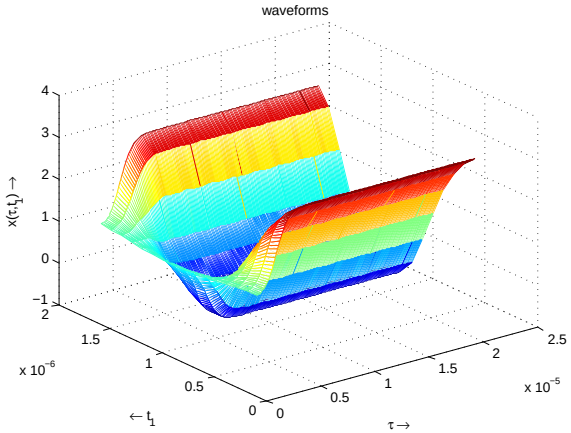
(c)

Figure :(a)Angular Frequency variations, (b) Dominating Waveforms (c) Solution along characteristic

Voltage Controlled Oscillators



(a)



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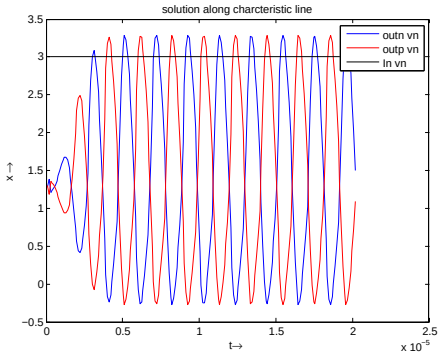

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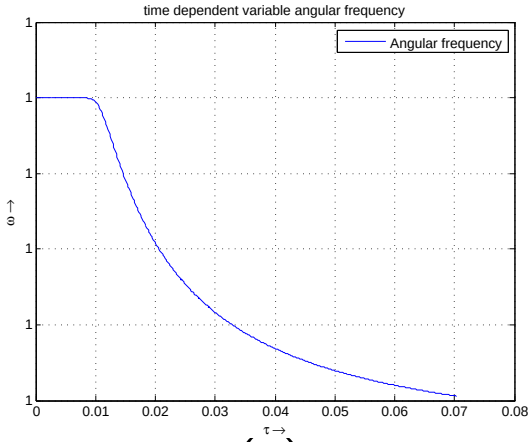
Figure :(a) Angular Frequency variations, (b) Dominating Waveforms (c) Solution along characteristic curve

Pierce Quartz Crystal Oscillators



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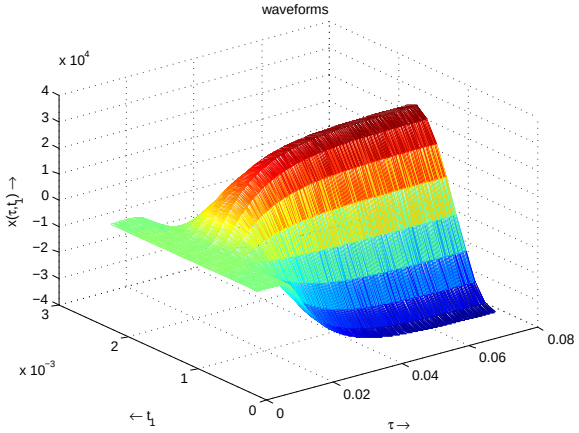

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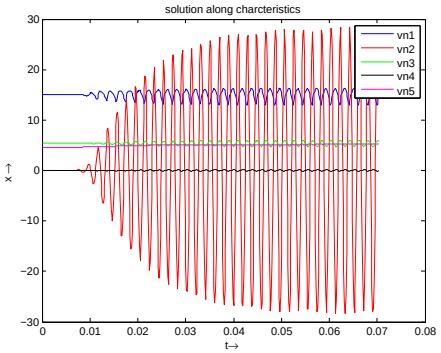

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Thank you