A Mixed Time Frequency Algorithm for Circuit Simulations

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Introduction

- At the high data rates requires huge signal bandwidths and high center frequency of several GHz led CAD tools to their limits.
- A novel method has been developed to circumvent Nyquist rate problem.
- The method is based on reformulating the ordinary DAE to a system of PDEs, also known as multirate PDEs (MPDEs).
- Formulation of PDE depends on the circuit class under investigation.
- The formulation of PDE also depends on number of fundamental tones or frequencies, therefore autonomous and non-autonomous circuits can be treated.
- Autonomous circuits comprise mainly oscillators, the frequency of oscillations is not known a-priori.
- The PDE formulation differs significantly from autonomous to non-autonomous case.
Consider the system of ordinary DAEs

\[
\frac{d}{dt} q(x)(t) = f(b(t), x(t)) \\
x_0 = x(t_0)
\]  

where, \( x : \mathbb{R} \rightarrow \mathbb{R}^N \), \( b : \mathbb{R} \rightarrow \mathbb{R}^N \) and \( f : \mathbb{R}^N \times \mathbb{R}^N \rightarrow \mathbb{R}^N \)

Introduce \( \hat{x} : \mathbb{R}^m \rightarrow \mathbb{R}^N \) for the state variables and \( \hat{b} : \mathbb{R}^m \rightarrow \mathbb{R}^N \) of the input signals.

For simplicity, we consider the function \( f, b, \hat{b} \in C^0 \) and \( q, x, \hat{x} \in C^1 \). In 1996, Brachtendorf et al. (Numerical steady state analysis of circuits driven by multi-tone signals, published in Elect. Eng.) have introduced the corresponding multirate partial differential algebraic equation (MPDAE)
Formulation

\[
\left( \frac{\partial}{\partial \tau} + \frac{\partial(\tau \omega_1(\tau))}{\partial \tau} \frac{\partial}{\partial t_1} + \ldots + \frac{\partial(\tau \omega_{m-1}(\tau))}{\partial \tau} \frac{\partial}{\partial t_{m-1}} \right) q(\hat{x}) \\
= f(\hat{b}(\tau, t_1, \ldots, t_{m-1}), \hat{x}(\tau, t_1, \ldots, t_{m-1})).
\]
Formulation

\[\left(\frac{\partial}{\partial \tau} + \frac{\partial (\tau \omega_1(\tau))}{\partial \tau} \frac{\partial}{\partial t_1} + \ldots + \frac{\partial (\tau \omega_{m-1}(\tau))}{\partial \tau} \frac{\partial}{\partial t_{m-1}}\right) q(\hat{x}) = f(\hat{b}(\tau, t_1, \ldots, t_{m-1}), \hat{x}(\tau, t_1, \ldots, t_{m-1})).\]

Important Result

A given solution \(\hat{x}\) of the MPDAE (3) coincides with a solution \(x\) of DAE (1) along the curve i.e. characteristic curve

\[x(t) = \hat{x}(t, \omega_1 t, \ldots, \omega_{m-1} t).\] (3)

In Radio Frequency (RF) applications, many system comprises exactly two different time scales:

MPDAE

\[\left(\frac{\partial}{\partial \tau} + \frac{\partial (\omega(\tau)\tau)}{\partial \tau} \frac{\partial}{\partial t_1}\right) q(\hat{x}(\tau, t_1)) = f(\hat{b}(\tau, t_1), \hat{x}(\tau, t_1))\] (4)
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Formulation of Algorithm

Selection of Initial Values

Cont...

\[ \hat{x}(0, t_1) = h(t_1) \quad \forall t_1 \in \mathbb{R}, \]  
(5)

\[ \hat{x}(\tau, t_1) = \hat{x}(\tau, t_1 + T) \quad \forall \tau, t_1 \in \mathbb{R}, \]  

\[ \hat{x}(\tau, t_1) \approx \sum_{k=-K}^{K} X_k(\tau) \exp(i\omega(\tau)kt_1) \]  
(6)

The Fourier coefficient \( X_k : \mathbb{R} \to \mathbb{C}^N \) with \( k = -K, \ldots, K \).
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**Formulation of Algorithm**

**Selection of Initial Values**

Cont...

\[ \hat{x}(0, t_1) = h(t_1) \quad \forall t_1 \in \mathbb{R}, \]  
\[ \hat{x}(\tau, t_1) = \hat{x}(\tau, t_1 + T) \quad \forall \tau, t_1 \in \mathbb{R}, \]  

\[ \hat{x}(\tau, t_1) \approx \sum_{k=-K}^{K} X_k(\tau) \exp(i\omega(\tau)k t_1) \]  

The Fourier coefficient \( X_k : \mathbb{R} \to \mathbb{C}^N \) with \( k = -K, \ldots, K \).

**Initial values estimation**, Stephanie Knorr, Wavelet-Based Simulation of MPDAS in RF Applications

Ph.D. Dissertation, Univ. of Wup, 2007

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Amplifier Circuit

Figure: (a) Solutions along characteristic curve, (b) Waveforms for output voltage
Differential Flip-Flop Circuit

Figure: (a) Solutions along characteristic curve, (b) Waveforms for output voltage
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Test Examples

Non-Autonomous Circuits

Differential to Single Circuit

(a) Solutions along characteristic curve, (b) Waveforms for output voltage

Figure: (a) Solutions along characteristic curve, (b) Waveforms for output voltage
Colpitts Oscillators

Figure: (a) Angular Frequency variations, (b) Dominating Waveforms, (c) Solution along characteristic
Voltage Controlled Oscillators

Figure: (a) Angular Frequency variations, (b) Dominating Waveforms (c) Solution along characteristic curve
Pierce Quartz Crystal Oscillators

Figure: (a) Angular Frequency variations, (b) Dominating Waveforms (c) Solution along characteristic curve
Thank you