Wavelet-Based Simulation Technique in the Time Domain

Kai Bittner

University of Wuppertal

Petrov-Galerkin discretization

- Circuit equations

$$\tfrac{d}{dt} q\big(x(t)\big) + g\big(x(t), t\big) = 0, \quad t \in [0, T], \qquad x(0) = x_0$$

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- Approximate the solution as $x(t) = \sum_i c_i \, \varphi_i(t)$ with suitable ansatz functions φ_i.
- Discretization: $\sum_{i=0}^{n} c_i \, \varphi_i(0) = x_0$ and

$$F_\ell(\mathbf{c}) := \int_0^T \left(\frac{d}{dt} q\big(x(t)\big) + g\big(x(t), t\big) \right) \theta_\ell(t) \, dt = 0,$$

for $\ell = 1, \ldots, n$ and suitable test functions θ_ℓ

Wavelet approach

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- Test functions: $\theta_\ell = \chi_{[\tau_{\ell-1},\tau_\ell]}$

$$\chi_I(x) = \begin{cases} 1, & \text{if } x \in I, \\ 0 & \text{otherwise} \end{cases}$$

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- Switch between representations by fast wavelet transform.

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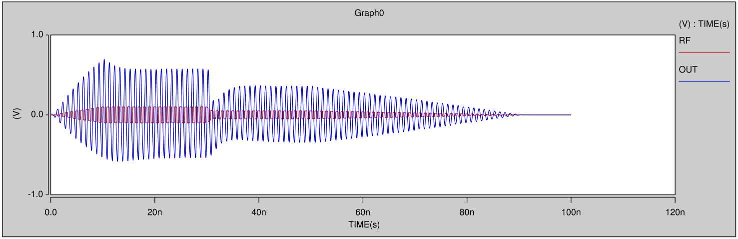
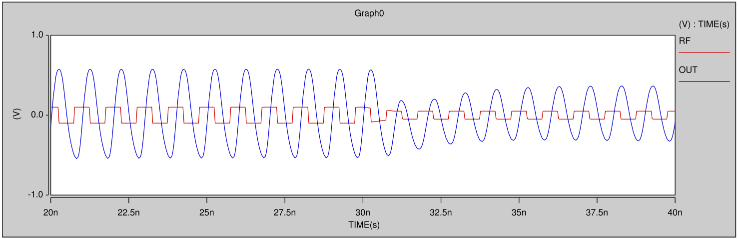
  $\rightsquigarrow$ Speedup of computation

Example — Amplifier (transAmp)



Input and output signal



Detail

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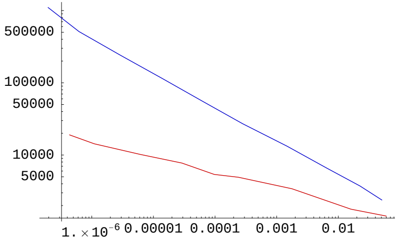
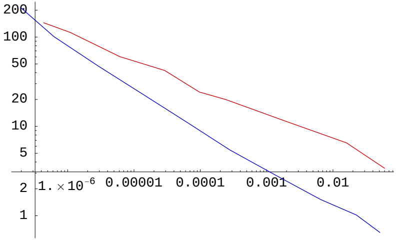


Grid size versus error



CPU time versus error

for transient analysis and subinterval wavelet method

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- Challenges:
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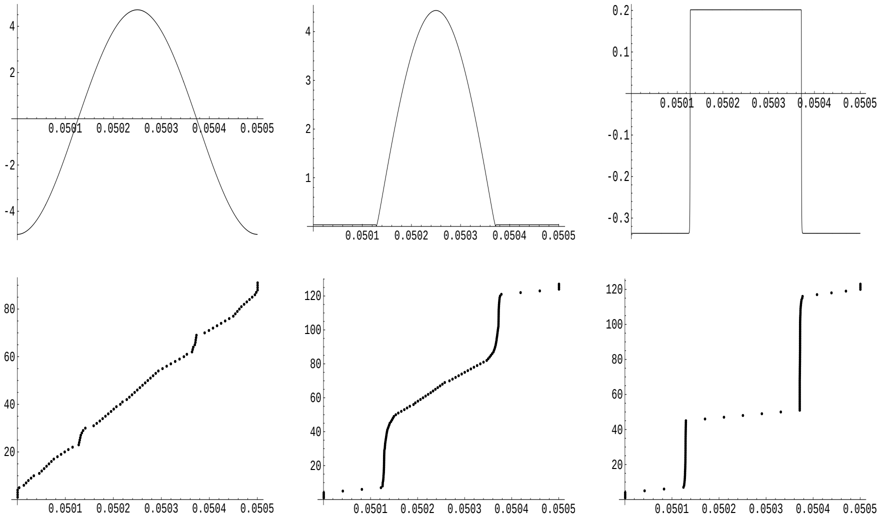
  - Balance the refinement of single components

Example — Schmitt trigger



Input, intermediate voltage and output, with corresponding grid

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Thank you

Questions?