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Initial Conditions for Harmonic Balance

WP2 – T2.3 – Jan ter Maten/NXP

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The Purpose

- **Distortion Analysis for free running oscillators**
  - Oscillators frequently occur on a chip: radio, gps, phone, Voltage Controlled Oscilators (VCOs), Ring-Oscillators.
  - Coupling / Immunity effects should be taken into account in the design process: shielding, 8-shaped-like inductors
  - Frequency ranges increase over the years (GHz).
  - Main distortion effect is a dynamic **Phase Shift** of the (time) **Periodic Steady-State Solution** (PSS)

    - via Harmonic Balance: M.M. Gourary, S.G. Rusakov et al [SCEE-2008],

    - via perturbation technique and differential equation for phase shift: D. Harutyunyan, J. Rommes, JtM, W. Schilders [IEEE TCAD 2009, 28(10), pp. 1456-1466]
      - Related to work done in WP1 (time domain)

The PSS Problem is a pre-requisite ➔ Task T2.3

- **Time domain integration**: Robust but can require a long time to converge.
  - Time-domain speed-up techniques: Poincaré-Map methods, Shooting.
- **Optimization techniques** [Lampe et al (2002,2003)]
- **Homotopy methods** [Welsch (1998), Brachtendorf (2002)]
- **Direct methods** :
  - time domain techniques: Finite Difference method (periodic boundary value problem)
  - frequency domain methods: Harmonic Balance
    - Freq f is additional unknown ➔ **Extra equation** (gauge) for settling phase
    - Newton methods become very sensitive with increase of f
      - Newton methods not easily deal with phase shift
    - **Require good initial estimate of f and of solution** (the last is the most difficult)
- Problem is a tough **nonlinear** eigen**value**/eigen**vector** problem.
  - Mathematics deals yet only with eigen**value** problems in which the eigenvalues have to solve a polynomial/rational expression [Mehrmann (2009), Meerbergen (2009)]

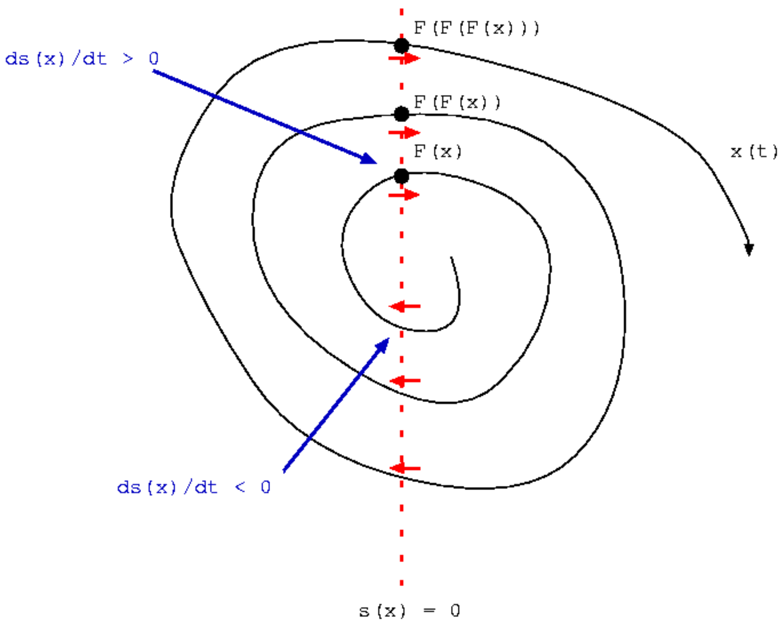
How to find Initial Estimates for f and for sol

- **T-domain integration & FFT** ➔ f & harmonics of nonlinear problem
- **Poincaré Map & Vector Extrapolation** [Houben, 2002-4]: needs info about final range of solution; but can be made adapted
- **AC-Sweep** ➔ f, first harmonic of linearised problem

- **Pole-Zero Analysis**: ➔ f
  - **Dominant Pole Algorithm, Rayleigh Quotient Iteration** ➔ f,x,y (right and left generalized eigenvectors of matrix pencil)
  - Can be applied to Monodromy matrix ➔ Floquet values and functions (for linear DAEs)

  - Note that u(t)=d/dt [x_{PSS}(t)] satisfies the homogeneous equations **linearised** around x_{PSS}(t)
    - u(t) is right Floquet function.
    - For gauging: use left Floquet function, because of bi-orthogonality
    - Newton Matrix of Finite Difference matrix (or HB) becomes (nearly) singular (using same stepsizes as for x_{PSS}(t))

Poincaré Map Method



Let $x_{n+1} = F(x_n)$, and $\Phi = \dfrac{\partial F}{\partial x}$, $x^* = F(x^*)$

Let $\Delta x_n = x_{n+1} - x_n$, then

$$\Delta x_{n+1} = \Phi(\Delta x_n) + O(\|\Delta x_n\|^2)$$

Let k be the smallest $k: \Delta x_k \in \mathrm{Span}(\Delta x_0, \ldots, \Delta x_{k-1})$

Then $\sum_{p=0}^{k} \sigma_p \Delta x_p = 0$, and with $\gamma = \sum_{p=0}^{k} \sigma_p$

$$x^* = \frac{1}{\gamma} \sum_{p=0}^{k} \sigma_p x_p + O(\|\Delta x_0\|^2)$$

Poincaré Map Method: Linear convergence

Minimal Polynomial Extrapolation ➔
Superlinear convergence

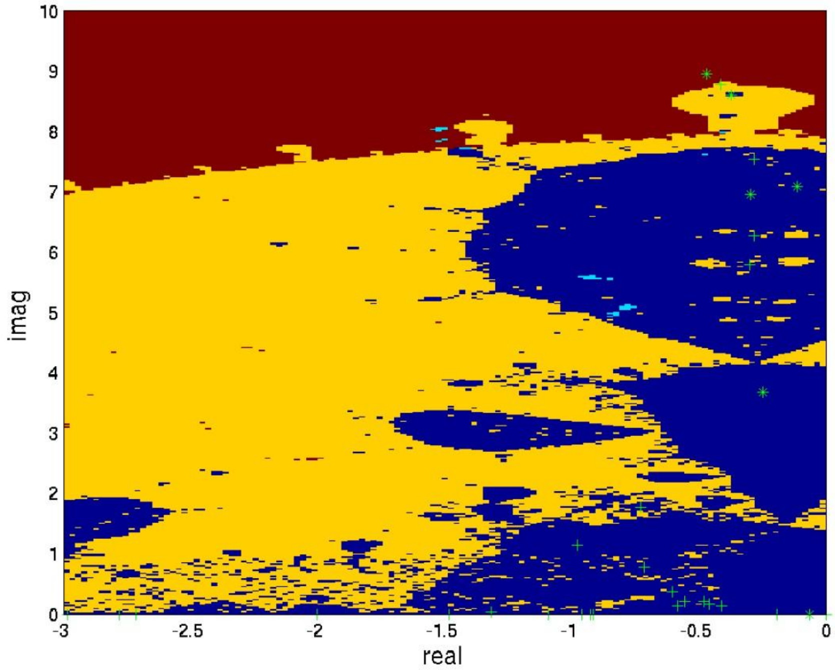
Assumes undamped time integration

Eigenvalue methods

- **Dominant Pole Algorithm (DPA, SADPA)**
  - Originates from Model Order Reduction [N. Martins, J. Rommes, 2006]
  - Converges quadratically to right-most generalised eigenvalue problem.
  - A subspace accelerated method converges with a selection criterion provides more poles used in a modal expansion of a transfer function.
  - Nice relation with sensitivity of eigenvalues wrt parameters

- **Rayleigh Quotient Iteration (RQI)**
  - Very similar to DPA
  - Converges cubically to eigenvalue nearest to shift.

- **Best method:**
  Hybrid method



Courtesy Joost Rommes

How to exploit? (Work in progress)

- Locally freezing [Liu - MSc/TUE, 2009]:
  - Partition time interval, freeze *C* and *G* matrices locally
  - Determine eigenvalues and eigenvectors on local interval
  - Determine local solution of time derivative, make continuous
  - Initial value of time derivative by 1 step time integration of *x*
  - Integrate time derivative ➔ *x*; make initial value periodic
- Alternative: Exploit that finally the FD-Newton matrix *Y* becomes singular
  - Write $Y_\mu = \lambda\, C(\mu,z) + G\,(\mu,z)$; first equation comes from periodicity (in *G*); *C* contains stepsizes or frequency (μ)
  - Determine generalized eigenvalue of pencil (*C*, *G*) closest to 1: λ. Let *x* be right eigenvector; periodicity equation in G ➔ periodic
  - λ ➔ rescale time interval (discretization stepsizes): $\mu = \mu\ |\lambda|$
  - *z := Int(x) + make periodic (ala waveform Newton)* ➔ Update matrices *C, G*
  - *Note that dz/dt can give a numerical approx. of x*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