Wavelets in Circuit Simulation

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Abstract Wavelet theory is a relatively recent area of scientific research, with a very successful application in a broad range of problems such as image, audio and signal processing, numerical analysis, electromagnetic scattering, data compression and denoising, stohastics, mathematics and physics, (bio)medicine, astronomy and many more. The key wavelet property contributing to its success in such a variety of disciplines is the capability of a simultaneous time and frequency representation of a signal embedded within a multi-resolution analysis (MRA) framework. The potential exploitation of this property for next-generation, wavelet-based techniques for analog circuit simulation is discussed in this paper.

1 Circuit simulation

Analog circuit simulation is a standard industry approach to verify an integrated circuit (IC) design at the transistor level before committing it to the expensive manufacturing process. An Electronic Design Automation (EDA) suite takes the circuit description originating from a designer's draft or fabrication data files, and automatically generates a network description in form of a text file called netlist, which describes circuit elements (resistors, capacitors, transistors, voltage and current sources, etc.) and their connections. Then a circuit simulator (SPICE and its derivatives), an integral part of an EDA suite, parses this input and translates it to a data format reflecting the underlying mathematical model of the system. This is done by applying the basic physical laws (energy and charge conservation) onto network topology and taking the characteristic equations for the network elements into account. The most used "translation" approach is the charge/flux oriented modified nodal analysis (MNA) [1], which yields a mathematical model in the form of an initial-value problem of differential-algebraic equations (DAEs):

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$$\mathbf{A}\frac{\mathrm{d}\mathbf{q}(\mathbf{x})}{\mathrm{d}t}+\mathbf{f}(\mathbf{x})=\mathbf{b}(t). \qquad (1)$$

The matrix $\mathbf{A}$ is called an incidence matrix and, in general, is singular. $\mathbf{x}$ is the vector of node potentials and specific branch currents. $\mathbf{q}$ is the vector of charges and fluxes. $\mathbf{f}$ comprises static contributions, while $\mathbf{b}$ contains the contributions of independent sources. A numerical solution to (1) is found using the Newton's method in combination with implicit time integration schemes and sparse matrix techniques.

Instead of describing the system with a minimal set of unknowns, the mathematical modeling of an electric network via the charge/flux oriented MNA approach aims to preserve the topological structure of the network [1], thus enabling a physical interpretation of simulation results by a user. Next, this approach preserves information on charge/flux conservation, a crucial property of many analog circuits like charge pumps, switched capacitor filters, etc. Furthermore, the charge/flux formulation enables more realistic modeling of nonlinear capacitors and inductivities. In addition, (1) is suitable for the usage of special integrator schemes such as multistep methods (BDF-Gear, Trapezodial rule) and it does not require second partial derivatives of charges resp. fluxes, which are usually not available in standard circuit simulation packages and may not even exist due to the lack of smoothness in modern transistor models. On the other hand, in general (1) is a stiff system, i. e. it involves characteristic time constants that differ by several orders of magnitude, which is a serious hindrance to obtaining accurate results in a reasonable amount of CPU time. In addition, this representation suffers from poor smoothness properties of modern transistor models [2], which are struggling to describe complex physical processes with the smallest possible set of mathematical equations. Furthermore, if more general models for network elements are utilized or refined models are used to include second order and parasitic effects, an ill-conditioned problem may arise and very special care must be taken to avoid divergence while finding a numerical solution to (1).

Today modern industrial analog circuit simulators are facing two serious challenges: qualitative and quantitative [1,3,4]. The *qualitative challenge* is highlighted when simulating circuits containing mixed analog-digital parts. At present there is no standardized framework within which is possible to simulate efficiently a mixed analog-digital circuit. Analog circuits to be simulated are often multitone oscillatory circuits, with widely separated carrier and modulation tones. A high-frequency carrier forces a small timestep while a low-frequency modulation forces a long simulation interval, resulting in unacceptable long simulation times even for moderately-sized RF circuits. Under the assumptions that the circuit behavior is periodic or at most quasi-periodic and that its frequency spectrum contains only a small number of frequencies, the multitone oscillatory circuits may be efficiently simulated using a specialized RF simulator based on either the frequency-domain Harmonic Balance or the time-domain Shooting algorithm [5]. However, a digital subpart in the circuit introduces a substantial amount of high-frequency components and the efficiency of these specialized solvers diminishes, if they can be applied at all. Hence the current approach to an IC design is to simulate the analog RF front-end in a specialized RF simulator, while the rest of the circuit is designed employing standard circuit simulation techniques. Due to this separation during the design process, subparts of mixed analog-digital circuits are usually not realized on the same die in order to keep spurious couplings between them as small as possible, since they cannot be easily characterized in a common simulation environment. However, with the trend towards ever-decreasing chip size, integration of analog and digital circuit parts on the same die is eminent and new simulation tools that can support these mixed designs are urgently needed.

The *quantitative challenge* lies in the simulation of extremely large circuits featuring several millions transistors, e. g. memory chips. The sheer size of the underlying MNA representation of such large circuits yields simulations that can last weeks, even longer than a month. Or they simply cannot be performed due to extreme memory and computational requirements. To cope with this situation, designers are forced to aggressively simplify these very large circuits and simulate only the most critical parts, an approach which is error prone. Or they use so called fast-SPICE simulators, which utilize speed-up techniques such as table look-up models, circuit partitioning, event-driven algorithms, hierarchical and parallel computations, etc. In this manner a fast-SPICE simulator is able to achieve a speed up of factor 1000 in comparison to a standard circuit simulator but at the price of reduced accuracy (usually as high as 3–5%), a mismatch that sometimes leads to sub-optimal designs and failure of produced ICs, thus necessitating expensive re-design cycles.

2 Introduction to wavelets

Wavelet theory emerged during the 20th century from the study of Calderon-Zygmund operators in mathematics, the study of the theory of subband coding in engineering and the study of renormalisation group theory in physics. The common foundation for the wavelet theory was laid down at the end of the 80's and beginning of the 90's by work of Daubechies [6,7], Morlet and Grossman [8], Donoho [9], Coifman [10], Meyer [11], Mallat [12] and others. Today wavelet-based algorithms are already in productive use in a broad range of applications [11–18], such as image and signal compression (JPEG2000 standard, FBI fingerprints database), speech recognition), numerical analysis (solving operator equations, boundary value problems), stohastics, smoothing/denoising data, physics (molecular dynamics, geophysics, turbulence), medicine (heart-rate and ECG analysis, DNA analysis) to name just a few. Recent approaches [19–23] to the problem of multirate envelope simulation indicate that wavelets could also be used to address the qualitative challenge by a development of novel wavelet-based circuit simulation techniques capable of an efficient simulation of a mixed analog-digital circuit.

A wavelet is a waveform of finite duration, with zero average value. Its shape is usually irregular and asymmetric, unlike sines and cosines in Fourier series representation. Nevertheless, just like sines and cosines in the classical Fourier expansion, wavelets may be used as basis functions for a wavelet expansion to represent electrical signals. The wavelet basis is formed via translations and dilations of a single wavelet function $\psi(x)$, called *mother wavelet*, according to

$$\psi_{s,\tau}(x) = s^{-1/2}\,\psi\left(\frac{x-\tau}{s}\right), \qquad (2)$$

where $(s,\tau) \in R^{+} \times R$. All wavelets from a specific basis are shifted (parameter τ) and dilated/compressed (by factor s) versions of this mother wavelet. The translation parameter τ is responsible for the localization in time of a corresponding wavelet. The scaling or resolution parameter s, usually called the scale, is generally understood as the frequency inverse. Therefore, the high scale (resolution) corresponds to low frequencies or a global view of the signal and low scale (resolution) corresponds to high frequencies or a detailed view of the signal. The factor $s^{-1/2}$ is used for energy normalization across different scales. From (2) it is clear that a wavelet basis intrinsically supports a *simultaneous* time-frequency representation of a signal, where the translation parameter τ is responsible for the time localization and the scaling parameter s for localization in the frequency domain. One particular wavelet property should be noted at this point: with wavelets it is not possible to exactly know a single frequency that exists at a single time instance, rather it is possible only to know what *frequency bands* exist at what *time intervals* [24].

There are numerous types of wavelets, each with different sets of features. Wavelets are usually grouped in wavelet families, according to several properties such as the support of wavelet and scaling functions, the number of vanishing moments, the symmetry, the regularity, existence of a scaling function ϕ, the orthogonality and biorthogonality, existence of explicit expression and others [13]. Some of the most famous wavelets families include: Haar, Daubechies, spline, biorthogonal, Morlet, Mexican hat, symlet, coiflet, Meyer, Bessel, Cauchy, Gaussian, etc.

Transforms involving wavelets can roughly be divided into three classes: continuous (CWT), discretised (DWT) and multi-resolution based (MRA). Contrary to the name, DWT is a continuous-time transform, as is CWT. The discreteness here refers to the fact that discrete wavelets are not continuously scalable and translatable functions but can only be scaled and translated in discrete steps determined by some integers (j,k). For example, a *discrete* wavelet suggested by Daubechies [7] is

$$\psi_{j,k}(x) = 2^{-j/2}\,\psi(2^{-j}x-k). \qquad (3)$$

DWT in combination with MRA is a very efficient transform with its linear computational complexity $\mathcal{O}(N)$, it is even more efficient than the Fast Fourier Transform (FFT) with its $\mathcal{O}(N\log N)$ complexity. Against the background of the circuit simulation, MRA is of particular interest and it will be further explored in more details.

2.1 Multi-resolution analysis

Formally defined, a *multi-resolution analysis (MRA)* in $L^2(R)$ is a set of closed subspaces V_s with $s \in Z$ such that the following five properties are satisfied [25]

1. $\ldots V_{-1} \subset V_0 \subset V_1 \subset \ldots \subset L^2(R)$, that is $V_s \subset V_{s+1}$ for all $s \in Z$
2. $\bigcup_{s=-\infty}^{+\infty} V_s$ is dense in $L^2(R)$; and in addition $\bigcap_{s=-\infty}^{+\infty} V_s = \{0\}$
3. $f(t) \in V_s$ iff $f(2t) \in V_{s+1}$
4. if $f(t) \in V_0$, then $f(t-k) \in V_0$ for all $k \in Z$
5. $\exists$ scaling function $\phi(t) \in V_0$, so that set $\{\phi(t-k) \mid k \in Z\}$ is a Riesz basis of V_0

The first (structural) property states that subspaces V_s in MRA are nested and the information at the resolution level s is entirely included in the information at higher resolution level $s+1$. The second (resolution) property states that the V_s, $s \in Z$, cover $L^2(R)$, i. e. the approximation approaches any signal in the entire initial space $L^2(R)$ as more details are added, i. e. resolution goes to infinity. On the other hand, as more and more details are removed, i. e. resolution gets coarser, only constant functions are left. In a limit, only the zero function remains, since the functions are squarely integrable. The third (dilation) property states that all V_s are scaled (dilated) versions of the central space V_0. The fourth (translation) property states that translation of $f(t)$ for some k does not change its resolution, i. e. V_0 is integral translation-invariant. From the properties 3 and 4 it directly follows that if a function f is in V_0, then its scaled and translated version $f(2^j t-k)$ is in V_j, i. e. if $f(t) \in V_0$, then $f(2^j t-k) \in V_j$ for all $k \in Z$. Finally, the fifth property states that similarly to the function $e^{j\omega t}$ in Fourier analysis, there exists one function $\phi(t)$ which generates the basis functions for all V_s. More precisely, if we define $\phi_{s,k} = 2^{s/2}\phi(2^s t-k)$, then $\{\phi_{s,k}(t)\}_{k\in Z}$ forms a Riesz basis of V_s.

To obtain the required resolution in a representation of an arbitrary signal, a sequence of scaling function expansions with wavelets of successively higher resolutions are used within the MRA. Interestingly, only *one* scaling function $\phi(t)$, called father wavelet, and *one* wavelet function $\psi(t)$, called mother wavelet, are needed to construct complete basis sets for systems of function spaces.

2.2 The wavelet expansion

Let us now consider a wavelet expansion embedded in the MRA framework. We start by considering an electrical signal as a combination of a smooth background and fluctuations superimposed on it, as is done for electrical field representation [26]. At a given resolution level s the signal is approximated in V_s by ignoring all the fluctuations above this level in V_k with $k > s$. Let $f_s(t) \in V_s$ denote the approximation of a signal $f(t)$ at given level s. In order to get better approximation, the level is increased to $s+1$ and a new approximation is obtained by adding the details, denoted as $d_s(t)$ to the approximation on previous level, i. e.

$$f_{s+1}(t) = f_s(t) + d_s(t). \tag{4}$$

Equation (4) means that at the resolution level $s+1$ a signal $f(t)$ is approximated with $f_s(t)$ in the scale subspace V_s and $d_s(t)$ in the detail subspace W_s. The scale subspace V_s consists of functions that contain the signal information down to scale 2^{-s}. The members of the detail subspace $W_s = V_{s+1} \ominus V_s$ are differences $d_s(t) = f_{s+1}(t) - f_s(t)$ and it comprises the additional information regarding details on scales between 2^{-s} and $2^{-(s+1)}$. For best approximation in terms of V_s the difference $d_s(t) = f_{s+1}(t) - f_s(t)$ should be orthogonal to $f_s(t)$. This is convenient to assume but not necessary. Assuming orthogonality means that $W_s \perp V_s$ and

$$V_{s+1} = W_s \oplus V_s = W_s \oplus W_{s-1} \oplus V_{s-1} = \ldots = \sum_{i=0}^{i=S} W_{s-i} \oplus V_{s-S}. \tag{5}$$

Furthermore, any two detail spaces at different resolutions are orthogonal, and the detail space W_s is orthogonal to an approximation space $V_{s'}$, only when $s > s'$, i. e. when the detail space is at a higher resolution level.

If the improvement of approximation (4) was continued to infinity, the original signal $f(t)$ would be recovered as:

$$f(t) = f_s(t) + \sum_{j=s}^{\infty} d_j(t). \tag{6}$$

Hence an arbitrary electrical signal expanded as a summation of scaling and wavelet basis functions may be denoted in a hierarchical manner as:

$$f(t) = \sum_{i=-\infty}^{s} c_i \phi_i(t) + \sum_{j=s}^{+\infty} \sum_{k=-\infty}^{+\infty} d_{j,k} \psi_{j,k}(t). \tag{7}$$

The first term in (7) is the projection of $f(t)$ into the scaling subspace V_s. It corresponds to a coarse approximation of $f(t)$ at a previously selected resolution level s. The second term consists of projections of $f(t)$ into the wavelet subspaces W_k.

In practical computations only finite sums can be used and hence the sums in (7) must be truncated. In general, we are interested in the behavior of the circuit over a certain finite time interval of length L. This implies that the upper limit of a sum in the first term (index i) and the inner sum of the second term (index k) would naturally depend on the interval considered, i. e. the parameter L. The outer sum of the second term (index j) defines the number of levels of detail that are to be taken into account, and hence the resolution level of the approximation will be defined by the upper boundary of this sum. For example, a finite approximation of an electrical signal over the time interval $[0,L]$ on a J^{th} resolution level could be denoted as:

$$f(t) \approx \sum_{i=0}^{2^s L-1} c_i \phi_i(t) + \sum_{j=s}^{(J-1)} \sum_{k=0}^{(2^j L-1)} d_{j,k} \psi_{j,k}(t) \tag{8}$$

At each resolution level j there are $2^j L$ basis functions, thus there are in total $(2^{J-s})L$ wavelet coefficients to be computed. In addition, there are $2^s L$ coefficients corresponding to scaling functions at a resolution level s. Hence the total number of coefficients in a finite wavelet expansion (8) over the interval $[0,L]$ on a J^{th} resolution level sums up to $2^J L$. For *efficient* computations the resolution level s should be chosen so that the coarse level is satisfied for most values of t and more details, i. e. wavelets, are added only at the points where they are needed to capture the abrupt signal fluctuations.

3 Wavelets in circuit simulation

Recent investigations into the use of wavelets in simulation of electronic circuits [19–23] have shown that these intrinsic properties make wavelets a natural candidate for a successful successor of time-domain (e. g. transient analysis, shooting analysis) and/or frequency domain (e. g. Harmonic Balance analysis) paradigms used in circuit simulation today. For example, Zhou and Cai propose the use of the wavelet collocation method in the time-domain [19] and the frequency domain [27] circuit simulation of mostly-linear circuits. For the computation of periodic steady state Soveiko and Nakhla [20,28] advocate a wavelet technique in combination with the Harmonic Balance approach, while Li et al. [29] use wavelet balance method. Christoffersen and Steer [21] used wavelets for transient circuit simulation within a state-variable based approach. Dautbegovic and Condon [22] use multitime partial differential equations (MPDE) in combination with wavelets for efficient simulation of multirate nonlinear RF circuits. Although valuable as a proof-of-concept, unfortunately these algorithms are still not mature enough to be used in industrial design flows.

We propose a wavelet expansion (8) embedded in the MRA framework as an approach to take when developing wavelet-based circuit simulation techniques. Consider the electrical signal depicted in Fig. 1, which is a typical time-domain output signal of a ring oscillator featuring a large amount of digital content. It can be considered as a “sum” of a digital signal and some irregular analog fluctuations. To describe such a signal efficiently, some sort of an adaptive approximation is needed. In such approximation an expansion of an electrical signal in those intervals where the signal varies smoothly and slowly should be simple and with as little degrees of freedom as possible, but whose resolution could be easily increased in places where the signal changes quickly and abruptly. For example, the smooth part could be represented by the low-resolution expansion of the signal, capturing the *average* signal behavior. A quickly changing part or *details* can only be captured by high-resolution components.

The wavelet expansion (8) is exactly the kind of the adaptive approximation that we are looking for. Embedded in the MRA framework, scaling functions can be used for an expansion of an electrical signal at a lower resolution level in those intervals where the signal varies smoothly and slowly, but in places where signal changes are quick and abrupt more details (i. e. wavelets) should be added. Therefore, the approximation effort is considerably reduced since only the “troublesome” regions are treated on a high-resolution level (i. e. with a larger number of coefficients), while smooth regions described on lower levels are captured by a smaller set of (possibly only) scaling coefficients. Compared to time-domain transient analysis,

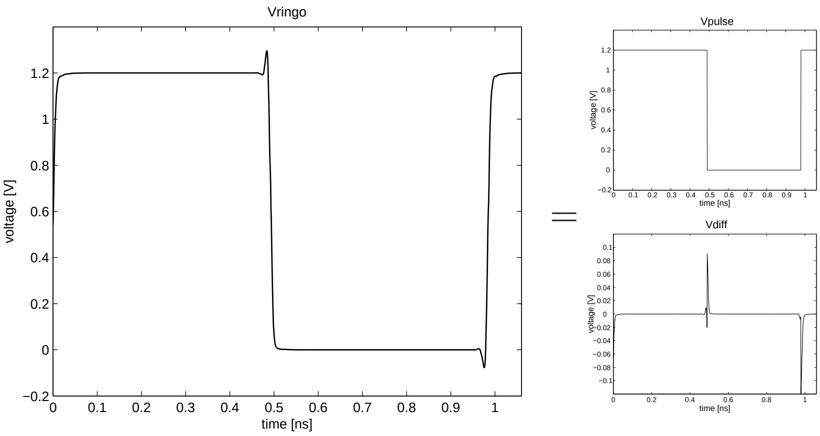

Fig. 1 An output voltage of a 1 GHz ring oscillator

taking fewer coefficients for the wavelet expansion in smooth regions is analogous to taking fewer time-steps during the transient analysis in intervals in which no large changes in signals are detected.

3.1 Advantages of the wavelet-based approach in circuit simulation

Let us now explore particularly advantageous properties of the wavelet expansion against the target application of circuit simulation.

Time-frequency representation. The truncated wavelet expansion (8) may be written in general form as $f(t) = \sum_{I\in\mathscr{I}} \mathbf{a}_I(f)\ \Psi_I$, where Ψ_I comprises all scaling and wavelet basis functions and $\mathbf{a}_I$ are the corresponding expansion coefficients on a finite index set $\mathscr{I} \leftrightarrow (j,k)$. In fact, these basis functions are generated by scaling (determined by the value of j) and translating (determined by the value of k) a single function ψ, i. e. $\psi_{j,k} = 2^{j/2}\ \psi(2^j t - k)$. Such an expansion associates with a function f, the array of coefficients $\mathbf{a} = \{\mathbf{a}_I(f)\}_{I\in\mathscr{I}}$ as is the case for the classical expansions. However, the coefficients $\mathbf{a}_I$ convey very detailed information on f due to the structure of $\mathscr{I}$ [30]. Each $\mathscr{I}$ comprises two-fold information on time (spatial) location encoded by k and information on scale, determined by j. Furthermore a scale is closely related to a frequency band and can be thought of as its inverse. Therefore, *each coefficient in a wavelet expansion (8) carries simultaneously both the time-domain and the frequency-domain information.*

Adaptive resolution. In contrast to approximating the function f of a given operator equation on some mesh (of fixed highest resolution), wavelet based schemes aim to determine its representation with respect to a basis [30]. This means that during the solution process, wavelet based algorithms will track only those coefficients in the unknown array **a** that are the most significant for approximating f with as few as possible degrees of freedom. This property contributes immensely towards the efficiency of such algorithms.

In addition, an adaptive resolution equips a wavelet expansion with a natural way for an easy trade-off between required accuracy and reasonable simulation time. If the amplitude of a fast-changing fluctuation is below the noise-floor or the design process is in its early stages, when a designer is interested only in an average behavior of a designed IC, fluctuations above certain pre-defined cut-off level can be neglected. While a-priori definition of this cut-off level can be tricky with standard approaches, with wavelets it is a trivial task of setting the required resolution level s.

Furthermore, if the approximation is not satisfactory, we can continue with progressively increasing the resolution level, thus adding finer resolution details to the signal. Theoretically, by continuing this process to infinity resolution level, the signal will be exactly recovered just like for example in case of Taylor series expansion in the time domain or Fourier expansion in the frequency domain.

Mixed analog-digital simulation. As briefly discussed in Section 1, at present there is no simulation framework (neither in the time nor in the frequency domain) in which a mixed analog-digital circuit can be efficiently simulated. The reason for this is a considerable approximation effort needed to capture a signal corresponding to one circuit part type when simulated in a simulator suitable for the other circuit type. For example, when a digital signal is to be simulated in a frequency-domain analog simulator, well suited for the analog RF front-end simulations, an extremely large number of Fourier coefficients is needed to accurately describe falling/rising edges of a digital signal. This is due to the poor time-domain localization property of the frequency-domain Fourier representation. In contrast, only a small number of coefficients corresponding to appropriately chosen scaling functions should be needed to approximate the signal well everywhere except in short intervals of sharp transitions. For those and only for those short intervals, additional coefficients corresponding to wavelet functions at higher resolution levels are needed to obtain equivalent or better accuracy to the Fourier representation, but at significantly reduced computation cost.

Validity range. A Taylor expansion places strong demands on the regularity of f such as analyticity, while wavelet expansion is typically valid for a much larger class of functions such as squarely integrable ones. This means that it is only required that the series on the right-hand side of (7) converges in the corresponding norm. Consequently the space of functions describing an electrical signal only needs to be a space of squarly integrable functions. Hence, a wavelet expansion has a potential to reduce negative influence of poor smoothness of transistor models on numerical convergence. However, this can only be confirmed after extensive testing on the existing industry models is performed within a working prototype of a wavelet-method.

3.2 Challenges of wavelet-based algorithms

The foreseen advantages of the use of wavelet-based techniques in circuit simulation highlighted in Section 3.1 give us a solid justification for investing efforts for developing wavelet-based algorithms. However before an industry-wide exploitation of these techniques is possible, the following issues need to be addressed.

Size of the wavelet expansion. For a numerically effective wavelet method it is crucial to setup near-optimal wavelet expansions, so that only a small number of wavelet coefficients is needed for a signal representation. Unlike with the Fourier basis, in which the shape of a basis function is predefined and cannot be changed, wavelet basis functions can have many shapes, varying from smooth to highly irregular. A wavelet algorithm can be setup without having a priori knowledge on the type of the wavelet basis set to be used for signal representation. In fact, if a user has some previous insights about the expected results, drawn upon experience or on some prior simulation results, then a suitable wavelet set may be chosen prior to simulation start, as one of simulation parameters. For example, a smooth wavelet set could be chosen for ICs involving smoother functions and more irregular ones for digital-like signals. Matching a wavelet basis set to a signal shape to reduce the number of needed expansion coefficients is analogous to choosing the appropriate base frequency in the Fourier expansion to describe periodic signals with a minimum set of coefficients corresponding to the expected maximum harmonic in a signal's spectrum prior to the HB computations. In addition, an adaptive selection of expansion time points as well as both hard- and soft-thresholding techniques [31–34] can help to further decrease the size of a system to be solved.

Numerical considerations. Even with a near-optimal selection of the wavelet basis the total number of wavelet coefficients is still very large; it equals the number of circuit variables times the number of coefficients in the chosen wavelet expansion for each node. For an efficient wavelet method a critical issue is how to store and invert a huge but relatively sparse Jacobian matrix arising from a Newton method applied to solve this nonlinear system. The investigations are ongoing into a setup of wavelet Jacobian in a block-diagonal form, which does not require storing the complete Jacobian at any point and is also easy to invert. Furthermore, one needs to be aware that significant matrix conditioning problems can arise due to a poor smoothness of MOSFET models (modeling problem) as well as solving higher-index DAEs (topological problem) and take appropriate care to minimize their negative influence on a solution process.

Applicability and functional considerations. The Harmonic Balance algorithm is an efficient tool for analyzing *periodic* or at most quasi-periodic circuits, unfortunately its use on any other type of circuits is a priori excluded. No such limitation is envisaged with wavelet based techniques and they are universal in the sense that they may be applied to any type of circuits. However, it is obvious that for pure sinusoidal signals there cannot exist a wavelet basis that is better than a Fourier basis in which a single expansion coefficient is needed to completely describe the signal.

But since the periodicity is not excluded from wavelet expansions, a wavelet basis can be found, such that it minimizes this expansion inefficiency and takes a small penalty when simulating pure sinusoidal circuits for the sake of generality.

Next, assuming that the previously mentioned challenges are successfully resolved and a wavelet solution is obtained, the question of the interpretation of these qualitatively new results arises. Wavelets are a powerful analysis tool but what can we conclude from a just performed wavelet analysis to enable a more robust design? An important point to enable faster adoption of wavelet based techniques in wider design community, governed by time- and frequency-domain specifications, is the derivation of a hopefully simple connection of wavelet-domain results to time- and frequency-domain design specifications.

4 Conclusion

With an ever-shrinking size and ever-increasing demand on functional complexity of a modern IC chip, a fast and scalable circuit simulation is a key design and verification approach in semiconductor industry. But increasing difficulties that current industrial circuit simulators are facing today, in particular in a simulation of mixed analog-digital circuit as well as circuits featuring millions of active devices, have highlighted the need for a novel approach to circuit simulation.

Intrinsic properties make wavelets a natural candidate for a successful successor of time- and frequency-domain paradigms used in circuit simulation today. This paper has discussed the advantages of wavelet expansions, which can be well utilized in circuit simulation, but also pointed out the challenges that must be resolved before an industry-wide acceptance and utilization of wavelet-based methods occurs. However, the expected benefits of a wavelet-based simulation engine, both in quantitative terms (efficient simulation of mixed-signal circuits) as well as qualitative terms (analyzing electrical signals with resolutions adapted to a problem at hands), is well worth allocating effort in a bid to develop the next-generation circuit simulators capable of answering industrial challenges of tomorrow.

Acknowledgements This work has been done within the ICESTARS (http://www.icestars.eu), a project supported by the EU Seventh Research Framework Project (FP7) with the grant number 214911.

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